

$$1- \text{Profit} = \text{Total Revenue} - \text{Cost}$$

$$2- \text{Total Revenue} = PD$$

P : Price D : Demand

$$3- \text{Cost} = C_{\text{fixed}} + C_{\text{variable}}$$

$$= C_F + C_U D$$

C_U = unit price

Steps: ① Find Profit

$$\text{② max Profit } D^* = \frac{d\text{Profit}}{dD} = 0, \quad \frac{d^2\text{Profit}}{dD} \text{ must be } -ve$$

$$\text{③ max Revenue } D^h = \frac{d\text{Revenue}}{dD} = 0, \quad \frac{d^2\text{Revenue}}{dD} \sim \sim \sim$$

④ Break even Point $\rightarrow$ 1st scenario (Profit 1st degree)

$\rightarrow$ 2nd scenario (Profit 2nd degree)

$\rightarrow$ Profit = 0

Profit above D

Profit below D_1, D_2

Note: - Price fun in Demand

- variable Cost Per mount CU , Per unit $\therefore CU = C_U D$

1. An electric power plant uses solid waste for fuel in the production of electricity. The cost Y in dollars per hour to produce electricity is $Y = 12 + 0.3X + 0.27X^2$, where X is in megawatts. Revenue in dollars per hour from the sale of electricity is $15X - 0.2X^2$. Find the value of X that gives maximum profit.

$$\text{Cost: } Y = 12 + 0.3X + 0.27X^2$$

$$\text{Revenue: } 15X - 0.2X^2$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$= 15X - 0.2X^2 - 12 - 0.3X - 0.27X^2 = -0.47X^2 + 14.7X - 12$$

$$\frac{d\text{Profit}}{dD} = -0.94X + 14.7 = 0$$

$$\therefore X = 15.63$$

$$\frac{d^2\text{Profit}}{dD} = -0.94 \quad \boxed{-ve}$$

$$\therefore \text{max Profit at } X = 15.63$$

2. A company produces and sells a consumer product, and is able to control the demand for the product by varying the selling price. The approximate relationship between price, p , and demand, D , is approximately

$$P = 38 + 2,700/D \quad \text{for } D \geq 1$$

Where p is the price per unit in dollars and D is the discrete demand per month. The company is seeking to maximum its net profit. The fixed cost is \$1,000 per month and variable cost is \$ D^2 per month.

- What is the optimal number of units that should be produced and sold each month to maximize profit?
- Show that your answer in part (a) maximizes profits.
- What is the unit price of your answer in part (a)?

$$P = 38 + \frac{2700}{D}$$

$$C_f = 1000$$

$$CV = D^2$$

$$\begin{aligned} a) \text{ Profit} &= TR - C_t = PD - (C_f + CV) \\ &= 38D + 2700 - 1000 - D^2 \\ &= -D^2 + 38D + 1700 \end{aligned}$$

$$\frac{d\text{Profit}}{dD} = -2D + 38 = 0$$

$$\therefore \text{max Profit at } D = 19$$

$$b) \frac{d^2\text{Profit}}{dD^2} = -2 = -ve$$

$$c) P = 38 + \frac{2700}{D} = 38 + \frac{2700}{19} = 180.105$$

3. A company has established that the relationship between the sales price for one of its products and the quantity sold per month is approximately $D = 780 - 10p$ units. (D is the demand or quantity sold per month, and p is the price per dollars.) The fixed cost is \$800 per month, and the variable cost is \$30 per unit produced.

- What number of units, D^* , should be produced per month and be sold to maximum net profit?
- What is the maximum profit per month related to the product?
- Determine D'_1 and D'_2 .

$$D = 780 - 10P \Rightarrow P \text{ must be fun in } D$$

$$P = 78 - \frac{1}{10}D$$

$$C_f = 800, \quad CV = 30$$

$$\begin{aligned} a) \text{ Profit} &= TR - C_t = PD - (C_f + CV) = PD - (C_f + CV D) \\ &= 78D - \frac{1}{10}D^2 - 800 - 30D \\ &= -.1D^2 + 48D - 800 \end{aligned}$$

$$\frac{d\text{Profit}}{dD} = -.2D + 48 = 0$$

$$\therefore \text{max Profit at } D = 240$$

$$\frac{d^2\text{Profit}}{dD^2} = -.2 = -ve$$

$$b) \text{ Profit} = -.1D^2 + 48D - 800 = 4960 \text{ \$}$$

$$c) D_1 = 462.71 \quad D_2 = 17.28$$

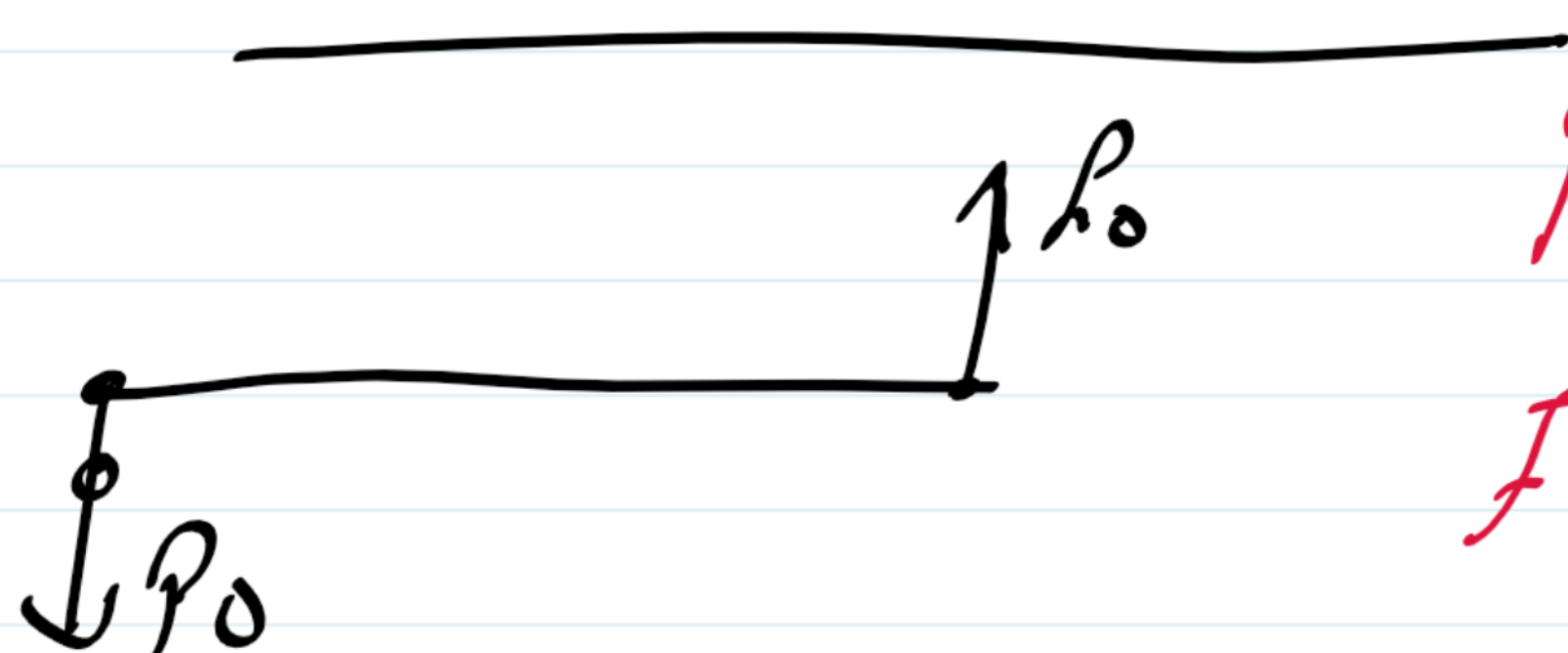
i : interest Rate n : num of Periods P : initial amount F : Final amount A : Series of equal Payment* Simple interest $(I) = P i n \Rightarrow$ يتعدى له كما يقول السبب

$$F = P + I$$

* Compound interest: $F = (1+i)^n P \Rightarrow$ Default

* Net flow: total inflow - total out

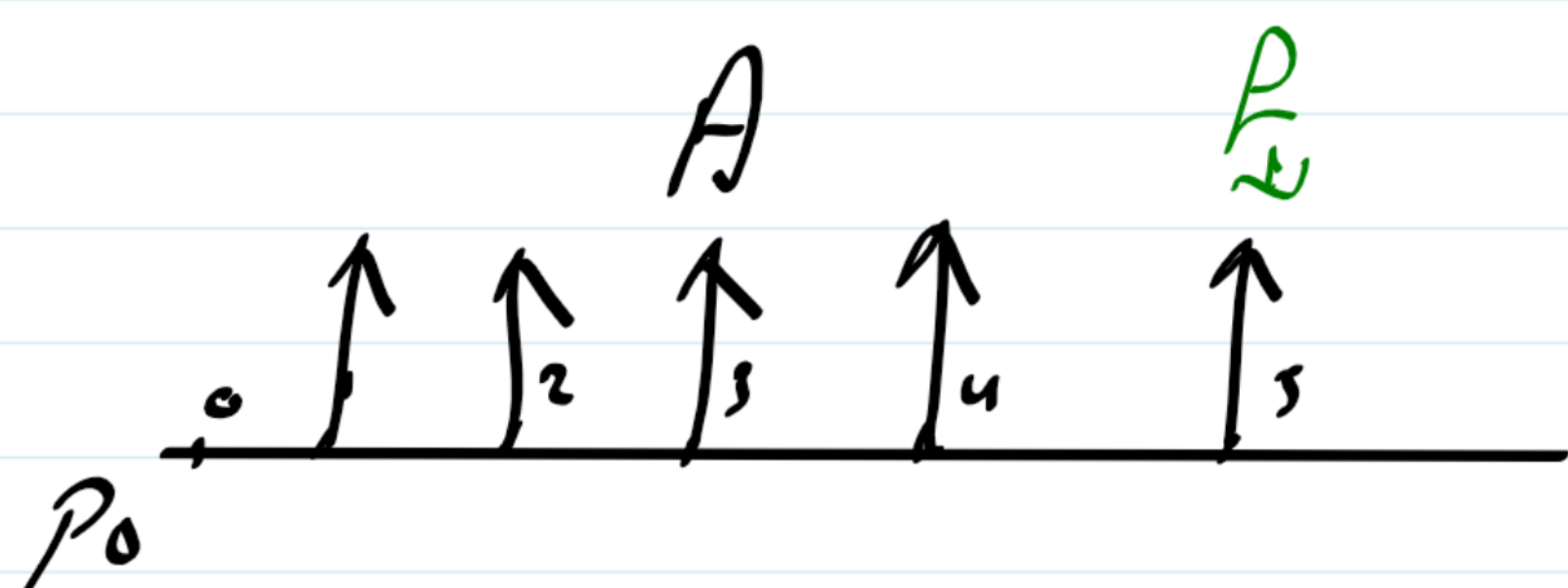
Simple Cash Flow



$$P = F (P/F, i\%, n)$$

$$F = P (F/P, i\%, n) = (1+i)^n P$$

* Series Cashflow

• Present (P_0) at first Payment - 1

$$\therefore t_0 - 1 = t_0$$

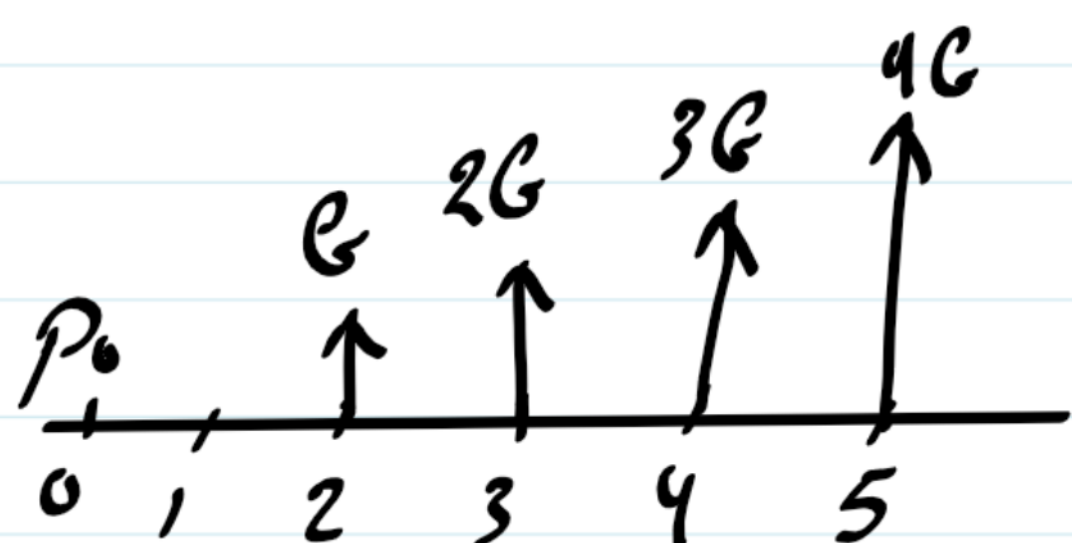
$$P = A (P/A, i, n)$$

$$F = A (F/A, i, n)$$

$$A = P (A/P, i, n)$$

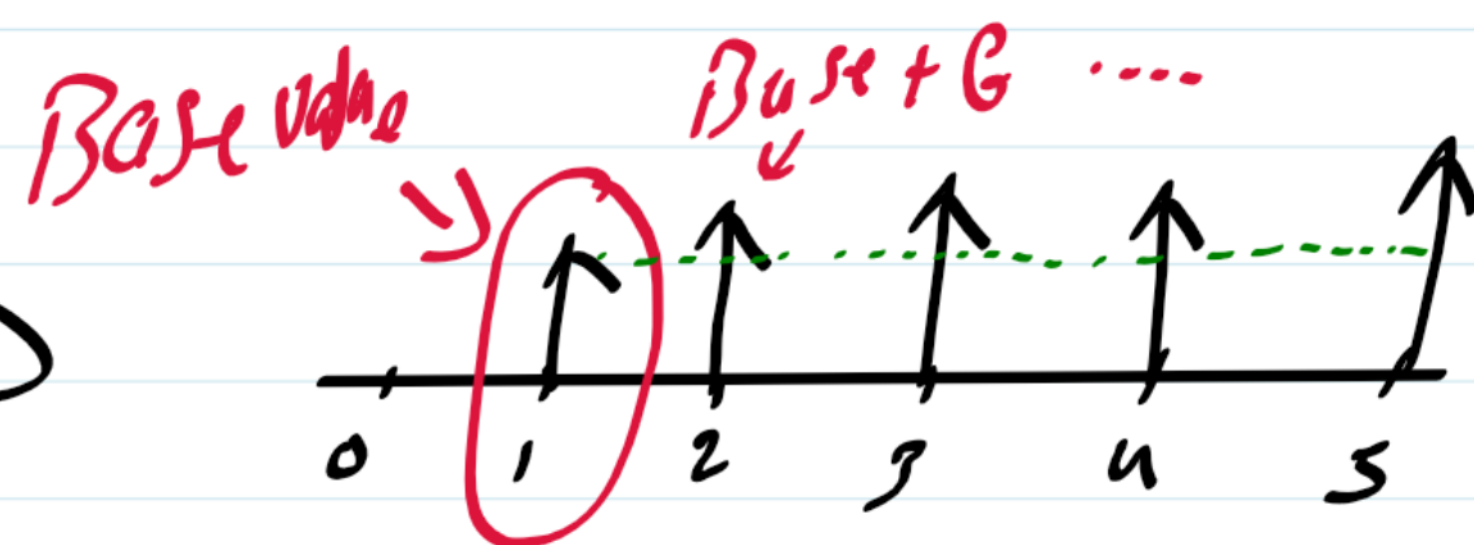
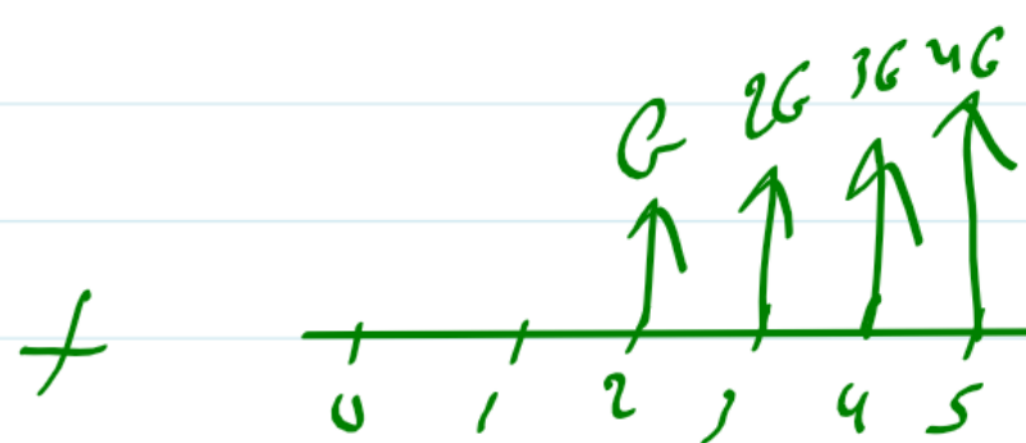
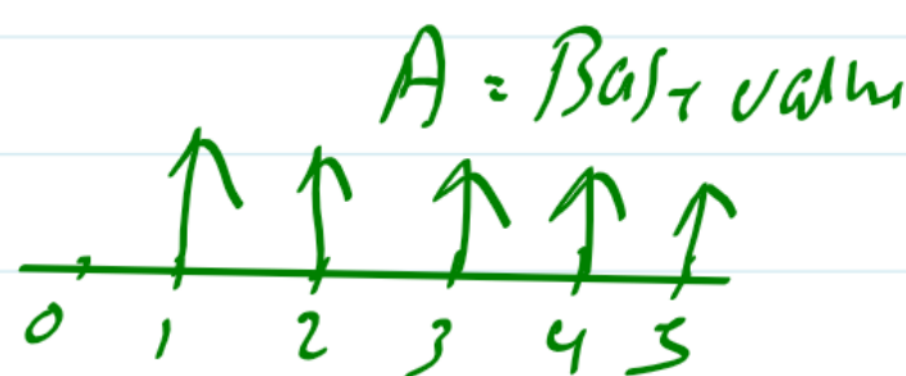
$$A = F (A/F, i, n)$$

* Gradient Cashflow:

Present P_0 at first Payment - 2

$$P = G (P/G, i, n)$$

$$G = P (G/P, i, n)$$

 $\Rightarrow$ 

interpolation:

$$i = 7.3\%, n = 10$$

$$b \left[\begin{matrix} a \\ 7.3 \\ 8 \end{matrix} \right]$$

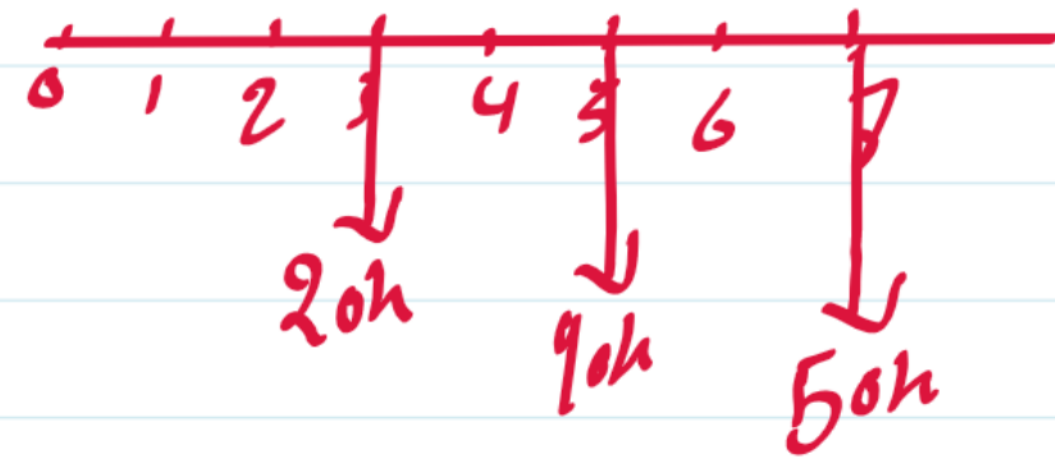
$$\left[\begin{matrix} .14238 \\ \lambda \\ .14903 \end{matrix} \right] c \quad d$$

$$c = \left(\frac{a}{b} \right) \cdot d = \left(\frac{7.3 - 7}{8 - 7} \right) (.14903 - .14238)$$

$$\lambda = c + .14238$$

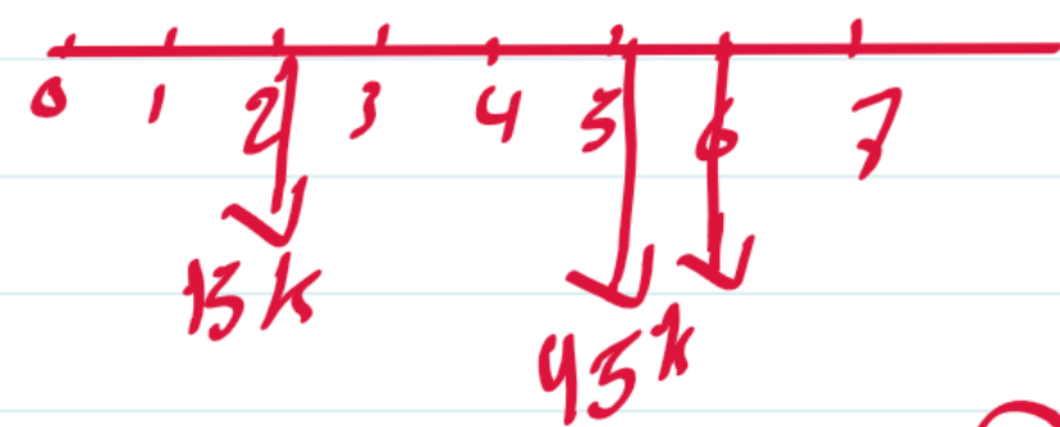
1. A factory owner wants to upgrade his production line. Two options of payments are afforded; the first is to pay LE 20,000, 40,000, 50,000 by the end of years 3, 5, and 7 from now. The second option is to pay LE 15,000, 45,000, 45,000 after 2, 5, and 6 years from now. If the annual interest rate is 8%, which option you recommend to the factory owner.

1st option:



$$\begin{aligned}
 P_0 &= 20k(P/F, 8\%, 3) + 40k(P/F, 8\%, 5) \\
 &\quad + 50k(P/F, 8\%, 7) \\
 &= 20k \times 0.7938 + 40k \times 0.6806 + 50k \times 0.5835 \\
 &= 72275 \text{ } \pounds
 \end{aligned}$$

2nd option:



$$\begin{aligned}
 P_0 &= 15k(P/F, 8\%, 2) + 45k[(P/F, 8\%, 5) + (P/F, 8\%, 6)] \\
 &= 15k \times 0.8523 + 45k[0.6806 + 0.6302] \\
 &= 71845.5
 \end{aligned}$$

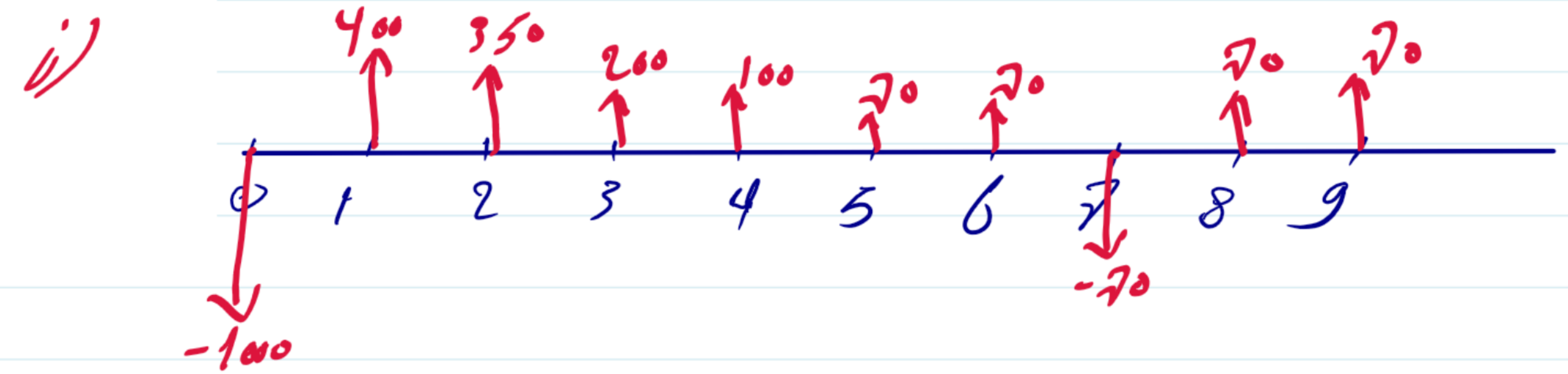
$\therefore$ 2nd option is better

2. Money transactions during the coming 9 years are given in the following table:

Year	0	1	2	3	4	5	6	7	8	9
Value LE	-1000	400	350	200	100	70	70	-70	70	70

The interest rate is 5% compounded annually.

- Draw the cash flow.
- Find the equivalent single payment at year 6.
- What should be the value in year 0 such that the present worth of the CF = 0?
- Convert the CF into 5 equal payments starting by the end of year 3.



ii)

$$\begin{aligned}
 P_0 &= -1000 + 400(P/F, 5\%, 1) + 350(P/F, 5\%, 2) + 200(P/F, 5\%, 3) + 100(P/F, 5\%, 4) \\
 &\quad + 70[(P/F, 5\%, 5) + (P/F, 5\%, 6) - (P/F, 5\%, 7) + (P/F, 5\%, 8) + (P/F, 5\%, 9)] \\
 &= 455
 \end{aligned}$$

$$F_6 = 455(F/P, 5\%, 6) = 610$$

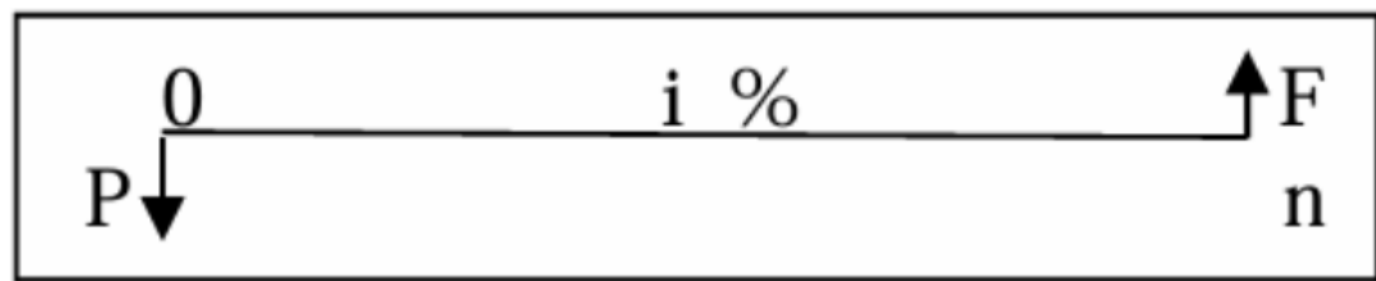
iii) $P_0 + X = 0 \quad \therefore X = -455$

iv) $F_2 = P_0(F/P, 5\%, 2) = 504.41$

$$A = P_2(A/P, 5\%, 5) = 115.825$$



3. Given the cash flow aside,

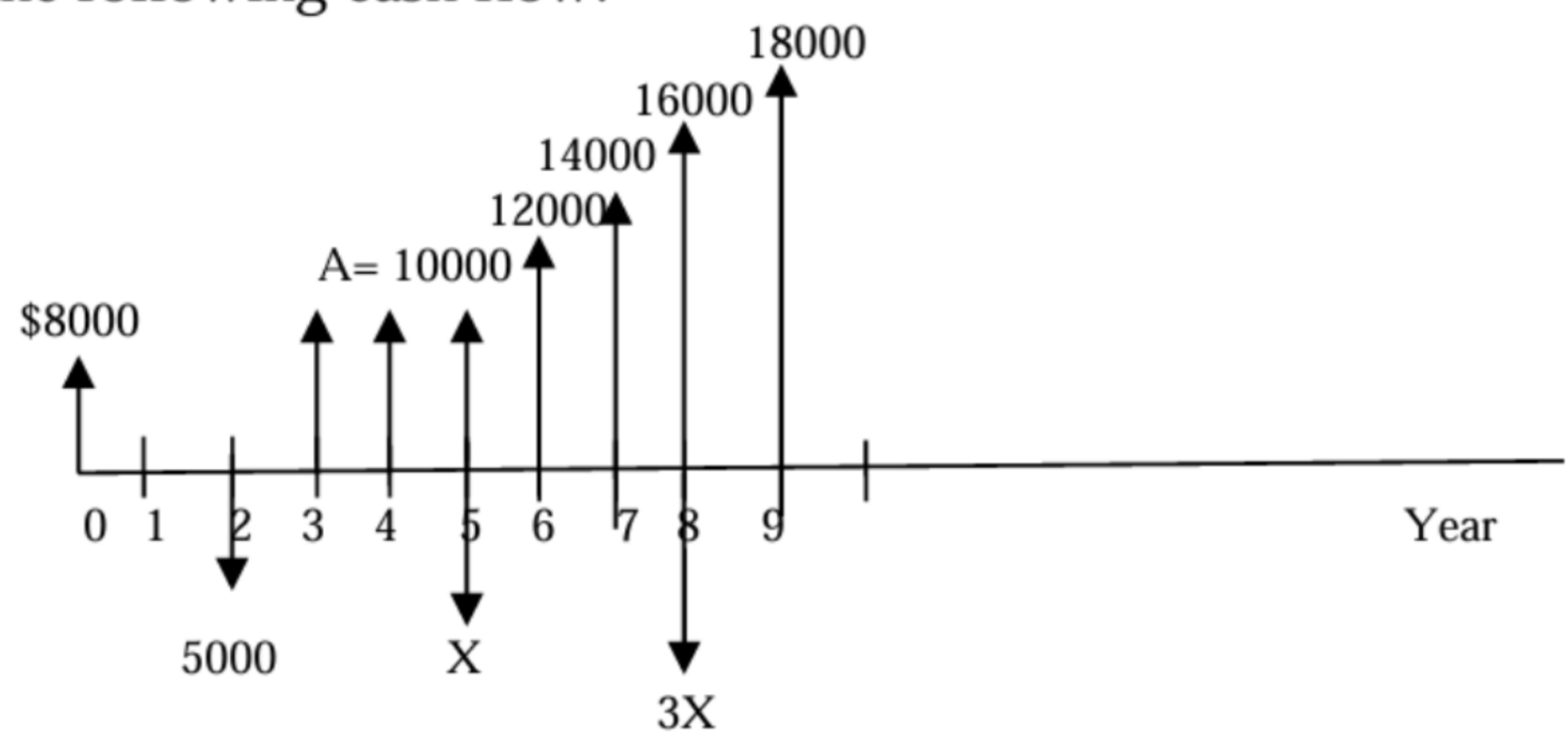


- If $P = \text{LE } 1000$, $F = 1500$, $n = 5$ years, find i .
- If $F = 2P$, $i = 8\%$, what is n ?
- If $F = \text{LE } 50,000$, $i = 7\%$, $n = 7$ years, what is P ?

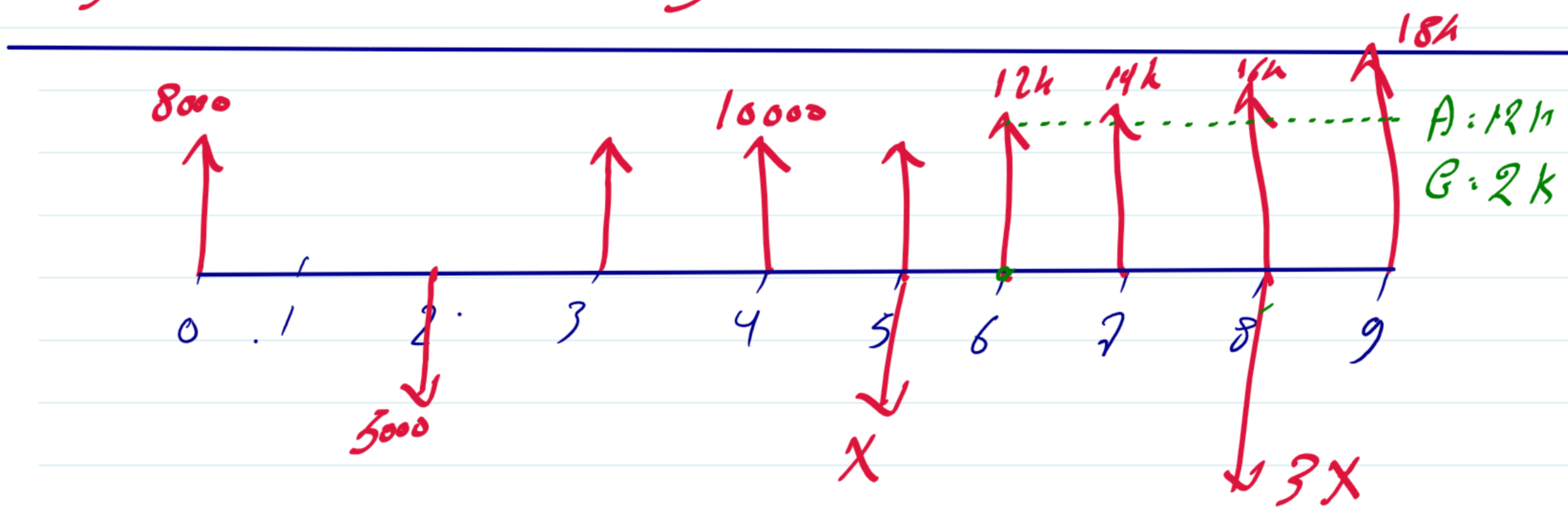
a) $F = (1+i)^n P \Rightarrow 1500 = (1+i)^5 1000 \therefore i = 8.4\%$

b) $F = (1+i)^n P \Rightarrow 2P = (1+0.08)^n P \therefore n = 9$

4. Given the following cash flow:



- Find the value of (X) such that the present worth of the cash flow is zero, given that the interest rate is 15% per year.
- Convert the CF into 5 uniform series starting at the end of year 2, given that $X = 0$.
- Given that $X = \$15,000$, Find the equivalent money value of the CF at year 12.



i) $P_0 = 8000 - 5000(P/F, 15\%, 2) + 10000(P/A, 15\%, 3)(P/F, 15\%, 2)$
 $+ 12000(P/A, 15\%, 4)(P/F, 15\%, 5) + 2000(P/B, 15\%, 4)(P/F, 15\%, 5)$
 $- X(P/F, 15\%, 5) - 3X(P/F, 15\%, 8) = 0$

$= 8000 - 5000 \times 0.7561 + 10000 \times 2.283 \times 0.7561 + 12000 \times 2.855 \times 0.4972 + 2000 \times 3.786 \times 0.4972$
 $- X \times 0.4972 - 3X \times 0.3269 = 0$

$\therefore X = 28608.25$

2nd method

$P_0 = 8000 - 5000(P/F, 15\%, 2) + \frac{10000}{15\%} (P/A, 15\%, 3)(P/F, 15\%, 2)$
 $+ 2000(P/B, 15\%, 5)(P/F, 15\%, 4) - X(P/F, 15\%, 5)$
 $- 3X(P/F, 15\%, 8) = 0$

$42272.55 - X(P/F, 15\%, 5) - 3X(P/F, 15\%, 8) = 0$
 $X = 28606.50$

ii) $A = P_0(F/P, 15\%, 1)(A/P, 15\%, 5)$
 $= 14503$

iii) at $X = 15000$ $P_0 =$
 $F_{12} = P_0(F/P, 15\%, 12)$

PROBLEM #1: (8 MARKS):

Table Q1 includes the money transaction values for an account over 20 years.

Mid 24

Table Q1: Money Transactions

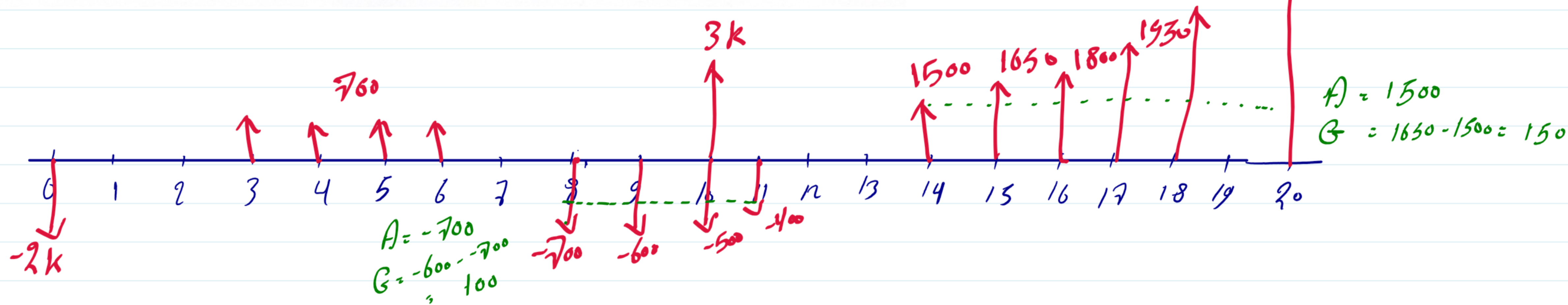
Time Period	0	3..6	T = 8..11	10	T = 14..18	20
Cash value	-£2000	700	-400 + 100 (T-11)	3000	1500+150(T-14)	5000

Given that the interest rate is 10% compounded annually,

- Construct the cash flow for the above money transactions.
- What is the equivalent value of the given cash flow at year 5?
- What is the equivalent annual value of the cash flow?

GOOD LUCK

a)



$$\begin{aligned}
 b) P_0 &= -2000 + 700 (P/A, 10\%, 4) (P/F, 10\%, 2) - 700 (P/A, 10\%, 4) (P/F, 10\%, 7) \\
 &+ 100 (P/G, 10\%, 4) (P/F, 10\%, 7) + 3000 (P/F, 10\%, 10) \\
 &+ 1500 (P/A, 10\%, 5) (P/F, 10\%, 13) + 150 (P/G, 10\%, 5) (P/F, 10\%, 13) \\
 &+ 5000 (P/F, 10\%, 20)
 \end{aligned}$$

$$\begin{aligned}
 &= -2000 + 700 \times 3.170 \times 0.8264 - 700 \times 3.170 \times 0.5132 + 100 \times 4.378 \times 0.5132 + 3000 \times 0.3855 \\
 &+ 1500 \times 3.791 \times 0.2897 + 150 \times 6.862 \times 0.2897 + 5000 \times 0.1486 = 2764
 \end{aligned}$$

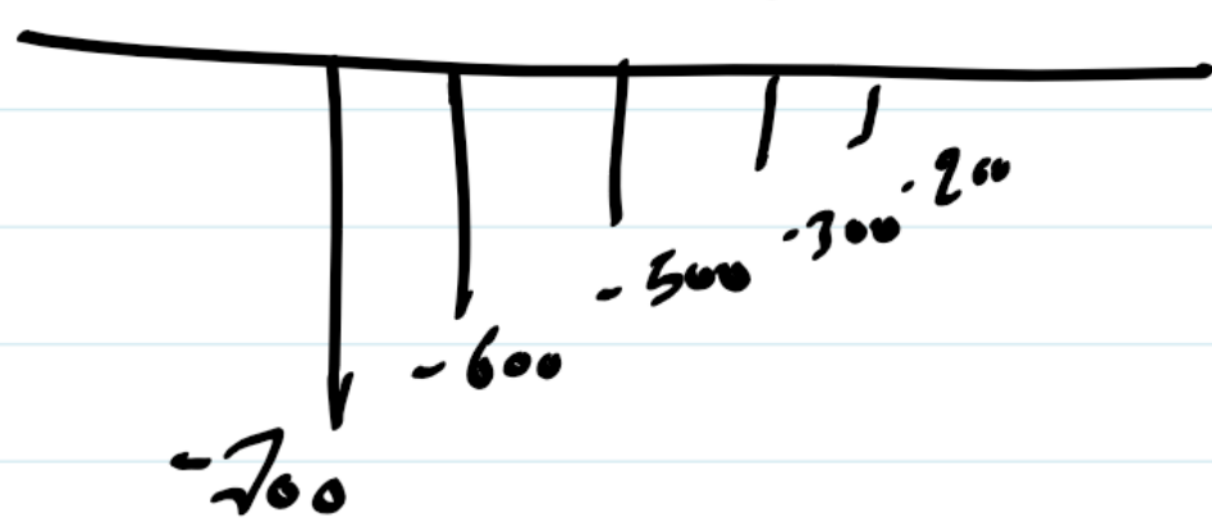
$$F_5 = P_0 (F/P, 10\%, 5) = 4452.804$$

$$c) A = P_0 (A/P, 10\%, 20) = 2764 \times 11.75 = 324.77$$

A: 1st term

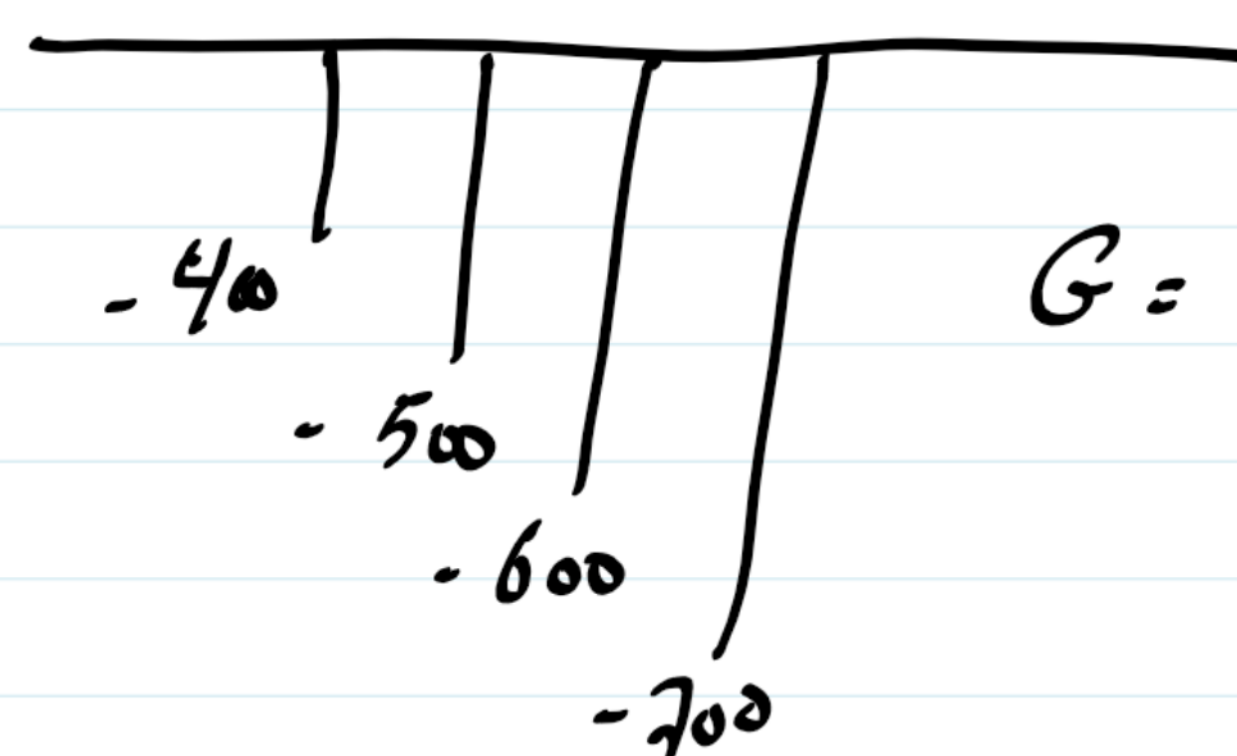
G: 2nd term - 1st term

* Negative gradient:



$$A = -700$$

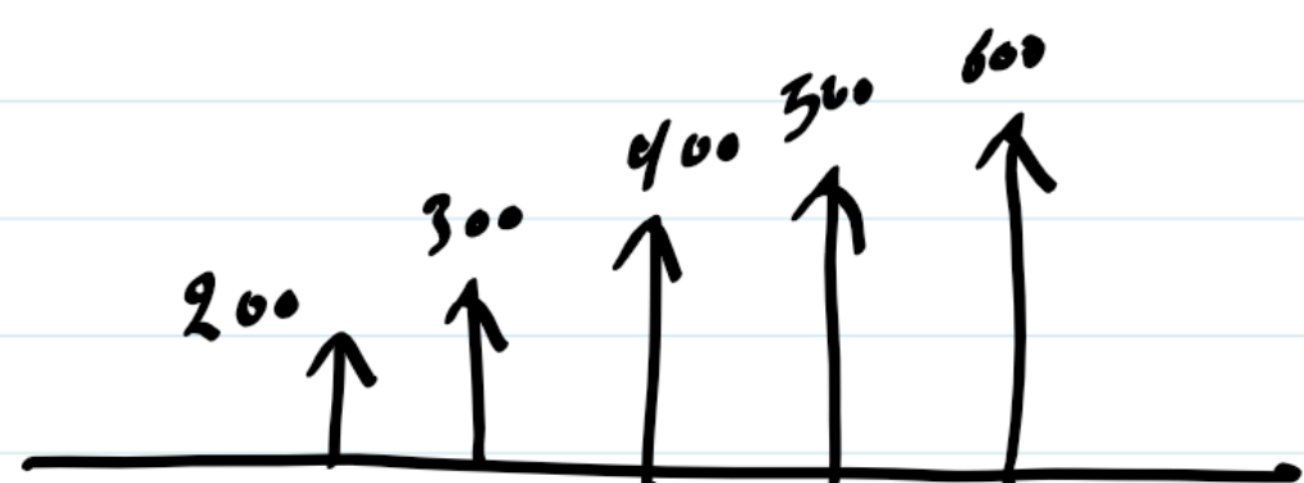
$$G = -600 - (-700) = 100$$



$$A = -400$$

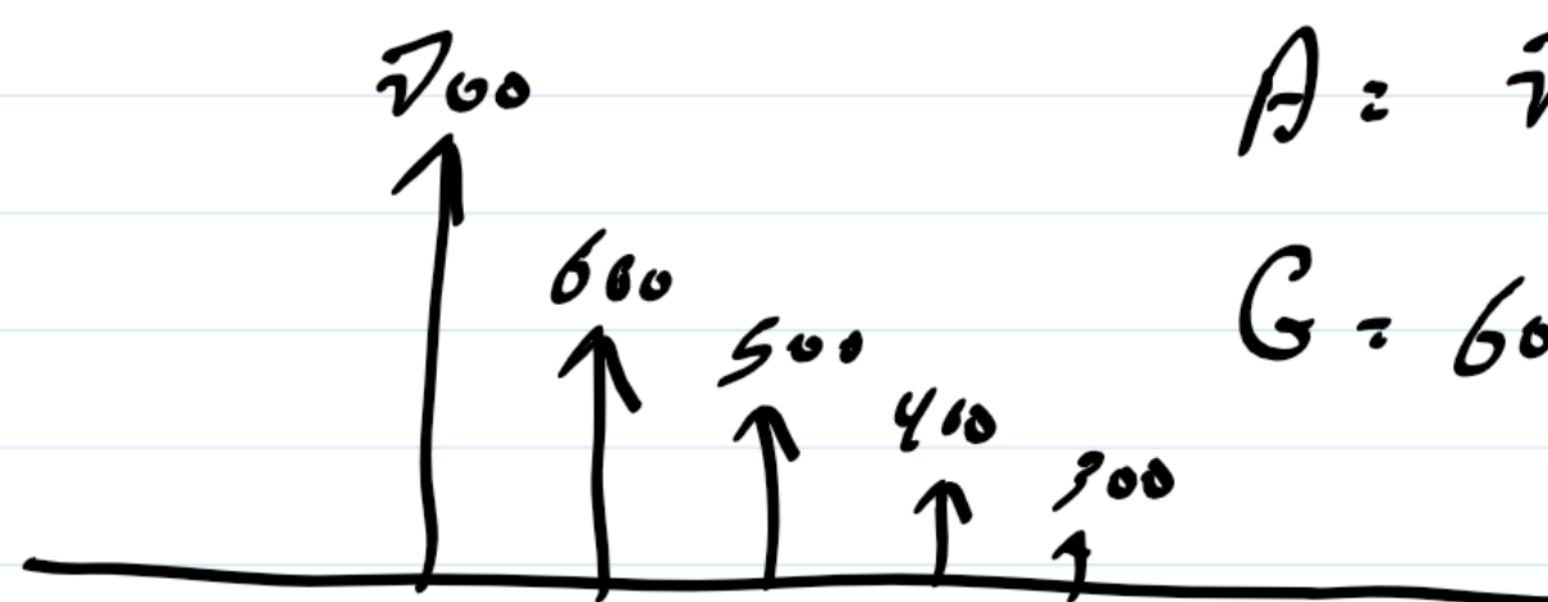
$$G = -500 - (-400) = -100$$

* Positive gradient



$$A = 200$$

$$G = 300 - 200 = 100$$



$$A = 700$$

$$G = 600 - 700 = -100$$

- a. A manufacturing company leases a building for \$100,000 per year for its manufacturing facilities. In addition, the machinery in this building is being paid for in installments of \$20,000 per year. Each unit of the product produced costs \$15 in labor and \$10 in materials. The product can be sold for \$40. How many units per year must be sold for the company to breakeven? (1.5 marks)

$$C_f = 100k + 20k = 120k$$

$$C_v = 15 + 10 = 25$$

$$P = 40$$

$$\begin{aligned} \text{Profit} &= TR - C_t \\ &= PD - [C_f + C_v D] \\ &= 40D - 120000 - 25D \\ &= 15D - 120000 \\ \therefore D &= 8000 \text{ unit} \end{aligned}$$

- b. Given that price per unit (\$) = $2,000 - D/10$, where D is the annual demand (units) and total cost per year can be approximated by $\$2,000 + 2D^2$. (4 marks)
- What level of annual demand maximizes profit? How do you know you haven't minimized profit?
 - What is the maximum profit (\$) per year?

$$P = 2000 - \frac{1}{10}D$$

$$C_t = 2000 + 2D^2$$

$$\begin{aligned} \text{a) Profit} &= TR - C_t = PD - C_t \\ &= 2000D - \frac{1}{10}D^2 - 2000 - 2D^2 \\ &= -2.1D^2 + 2000D - 2000 \end{aligned}$$

$$\frac{d\text{Profit}}{dD} = -4.2D + 2000 = 0 \quad \therefore D = 476.1 \approx 477$$

$$\frac{d\text{Profit}}{dD^2} = -4.2 = -ve \quad \text{2nd Derivative is negative} \therefore \text{Profit is maximized}$$

$$\text{b) Max Profit} = -2.1(477)^2 + 2000(477) - 2000 = 474139$$

- c. A postal service carrier works on Sunday delivering special high-priority letters. She can deliver 15 letters per hour. The hourly fixed costs (vehicle depreciation, insurance, etc.) equal \$42, and the hourly variable costs (time and a half wages, etc.) total \$0.50 per letter. How much must the postal service charge for each letter delivered in order to break even on Sunday deliveries? (1.5 marks)

$$\begin{aligned} \text{Profit} &= PD - C_f - C_v D \\ &= (15P - 42 - 0.5) \times 24 \end{aligned}$$

$$\text{Profit} = 0$$

$$\therefore P = 3.3 \$$$

$$D = 15 \text{ Per hour}$$

$$C_f = 42$$

$$C_v = 0.5$$

$$P = ?$$

$$\therefore D = 15 \times 24 = 360 \text{ Per day}$$

$$C_f = 24 \times 42 = \text{Per day}$$

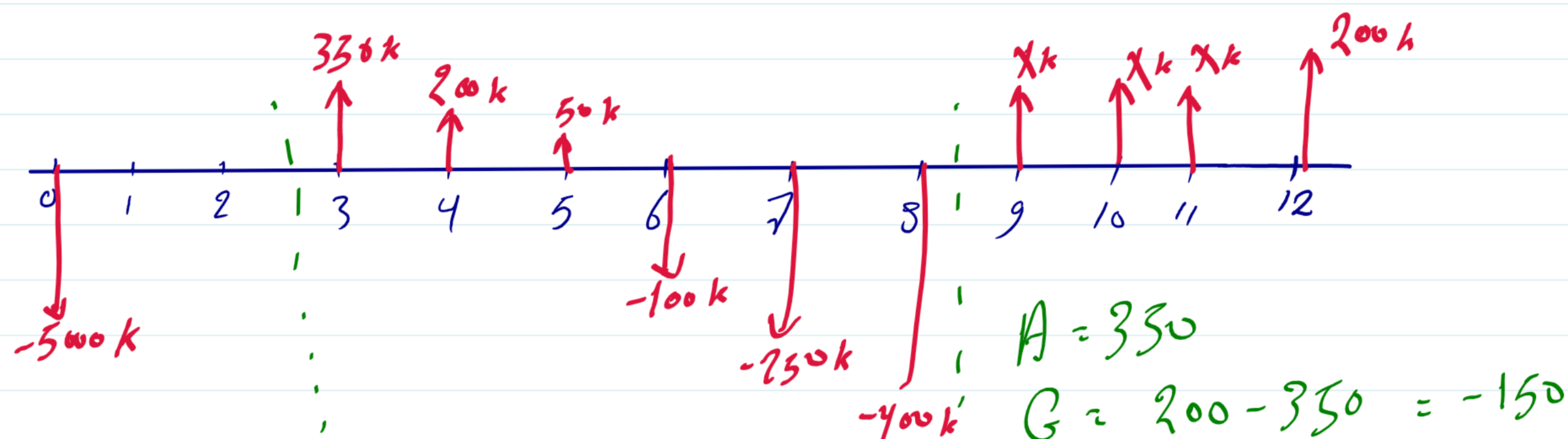
PROBLEM #1: (7 MARKS):

The data in the table below includes the money transactions over a 12-Year time period:

Time period T	0	3..8	9..11	12
Cash Flow (\$ 1000)	-5000	$-[400 + 150(T-8)]$	X	200

Using an annual interest rate of 10% compounded annually, answer the following:

- Draw the cash flow for the money transactions
- Determine the value of X that makes the equivalent amount of PW equals \$2000,000. → Present worth P_0



$$P_0 = -5000 + 350 (P/A, 10\%, 6) (P/F, 10\%, 12) - 150 (P/G, 10\%, 6) (P/F, 10\%, 12) + X (P/A, 10\%, 3) (P/F, 10\%, 18) + 200 (P/F, 10\%, 12) = \frac{2000000}{1000}$$

$$P_0 = -5000 + 350 \times 4.355 \times 0.8264 - 150 \times 9.684 \times 0.8264 + X \times 2.487 \times 0.4683 + 200 \times 0.3186 = 2000$$

$$\therefore X = 5927.55$$

- To maintain constant pressure on engine valves, and improve efficiency in automotive engine, lash adjuster is used. Equation below shows the link between the lash adjuster price (p) and monthly demand (D).

$$D = \frac{3000 - p}{0.15}$$

To maximize total revenue, what demand should be considered? What important data needed if maximum profit is desired? (4 marks)

$$P = 3000 - 0.15D$$

$$TR = PD = 3000D - 0.15D^2$$

$$\frac{dTR}{dD} = 3000 - 0.3D \quad \therefore \text{max Revenue at } \hat{D} = 10000$$

total cost is needed to find Demand for maximizing profit

b. A manufacturing company leases a building for \$100,000 per year for its manufacturing facilities. In addition, the machinery in this building is being paid for in installments of \$20,000 per year. Each unit of the product produced costs \$15 in labor and \$10 in materials. The product can be sold for \$40. (4 marks)

i. How many units per year must be sold for the company to breakeven? (2.5 marks)

ii. If 10,000 units per year are sold, what is the annual profit? (1.5 marks)

i)

$$\text{Profit} = TR - CT$$

$$= PD - [CF + CV D]$$

$$= 40 D - 120\,000 - 25 D$$

$$= 15 D - 120\,000$$

$$\therefore D = 8000 \text{ unit}$$

$$CF = 100k + 20k = 120k$$

$$CV = 15 + 10 = 25$$

$$P = 40$$

ii) Profit at $D = 10\,000$

$$\therefore \text{Profit} = 15 \times 10\,000 - 120\,000 = 30\,000$$