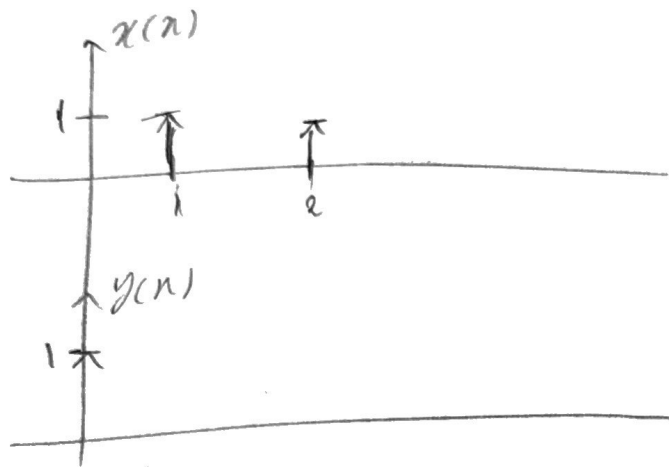


(Nov 2024 Mid)

$$(1) \quad x(n) = u(n) - u(n-3) = \delta(n-1) + \delta(n-2)$$

$$y(n) = u(n) - u(n-1) = \delta(n)$$



$$x(n) = \{0, 1, 1, 0\}$$

$$y(n) = \{1\}$$

$$x(n) * y(n) = x(n) * \delta(n) = x(n) = u(n) - u(n-3)$$

(2)

$$y(n) = 0.5 y(n-1) + -0.3 y(n-2) + x(n) + 0.4 x(n-1)$$

$$y(z) + z^{-1} 0.5 y(z) + -0.3 z^{-2} y(z) = X(z) + 0.4 z^{-1} X(z)$$

$$y(z) [1 + 0.5 z^{-1} + -0.3 z^{-2}] = X(z) [1 + 0.4 z^{-1}]$$

$$H(z) = \frac{y(z)}{X(z)} = \frac{1 + 0.5 z^{-1} - 0.3 z^{-2}}{1 + 0.4 z^{-1}} = \frac{z^2 + 0.5z - 0.3}{z^2 + 0.4z}$$

$$= \frac{z^2 + 0.5z}{z^2 + 0.4z} - \frac{0.3}{z^2 + 0.4z} = \frac{z(z + 0.5)}{z(z + 0.4)} - \frac{0.3}{z(z + 0.4)}$$

$$= 1 + \frac{0.1}{z + 0.4} - \left[\frac{0.75}{z} - \frac{0.75}{z + 0.4} \right] \frac{A}{z} + \frac{B}{z + 0.4} = \frac{0.3}{z}$$

$$= 1 + \frac{0.1}{z + 0.4} - \frac{0.75}{z} + \frac{0.75}{z + 0.4}$$

$\therefore z = 0.4, z = 0 \Rightarrow |z| < 1$ (under the unit circle)

The system is stable

①

$$A(z + 0.4) + Bz = 0.3$$

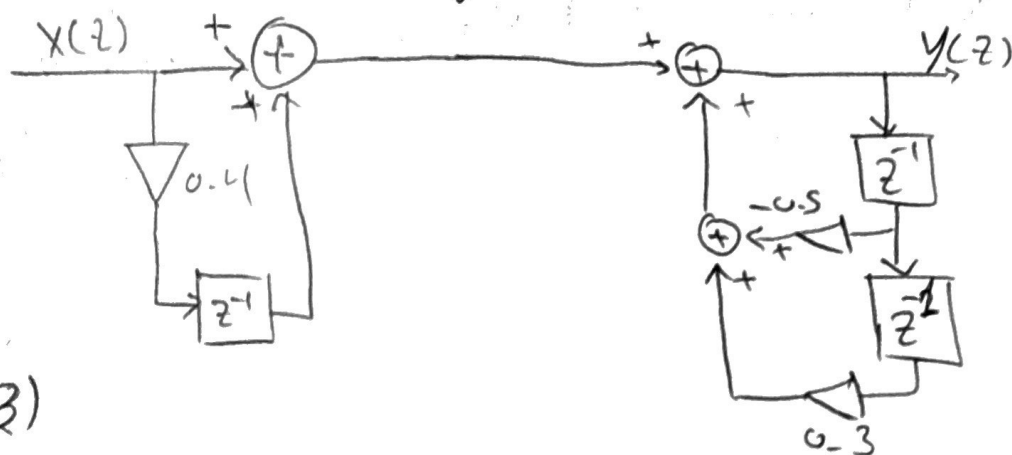
$$A z = 0$$

$$A = 0.75$$

$$A z = 0.4 B = -0.75$$

$$y(z) + 0.5z^{-1}y(z) - 0.3z^{-2}y(z) = x(z) + 0.4z^{-1}x(z)$$

$$y(z) = \frac{x(z) + 0.4z^{-1}x(z)}{1 - 0.5z^{-1} + 0.3z^{-2}}$$



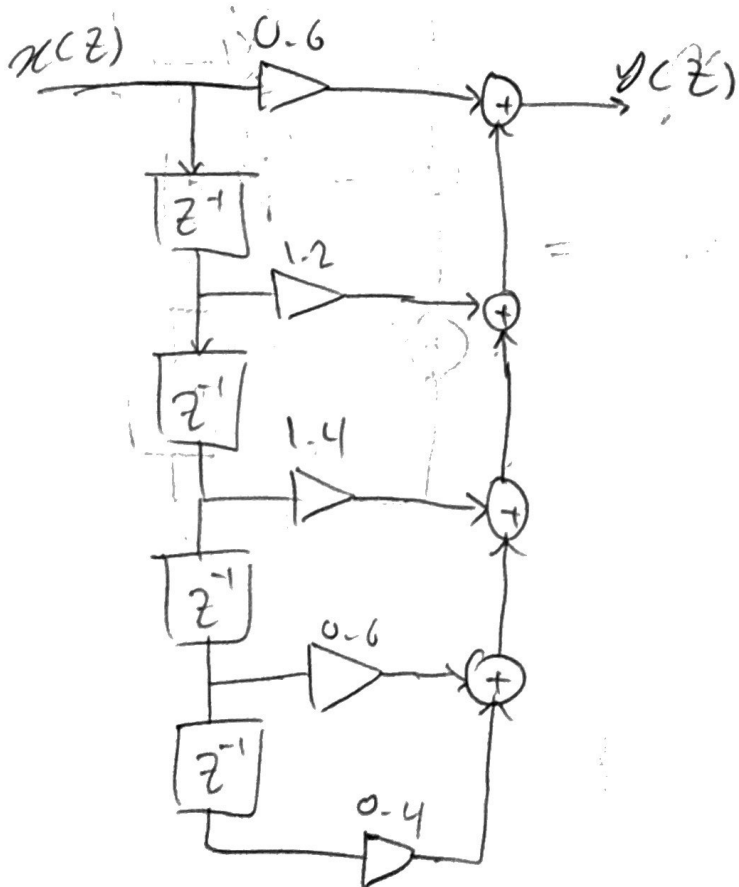
(3)

$$H(z) = 0.6 + 1.2z^{-1} + 1.4z^{-2} + 0.6z^{-3} + 0.4z^{-4}$$

$$\frac{y(z)}{x(z)} = \frac{0.6 + 1.2z^{-1} + 1.4z^{-2} + 0.6z^{-3} + 0.4z^{-4}}{1}$$

$$y(z) = 0.6x(z) + 1.2x(z)z^{-1} + 1.4x(z)z^{-2} + 0.6x(z)z^{-3} + 0.4x(z)z^{-4}$$

$$y(n) = 0.6x(n) + 1.2x(n-1) + 1.4x(n-2) + 0.6x(n-3) + 0.4x(n-4)$$



(2)

$$H(z) = 0.6 + 1.2z^{-1} + 1.4z^{-2} + 0.6z^{-3} + 0.4z^{-4}$$

$$H(e^{j\omega}) = 0.6 + 1.2e^{-j\omega} + 1.4e^{-2j\omega} + 0.6e^{-3j\omega} + 0.4e^{-4j\omega}$$

$$= \underline{CH(\omega)}$$