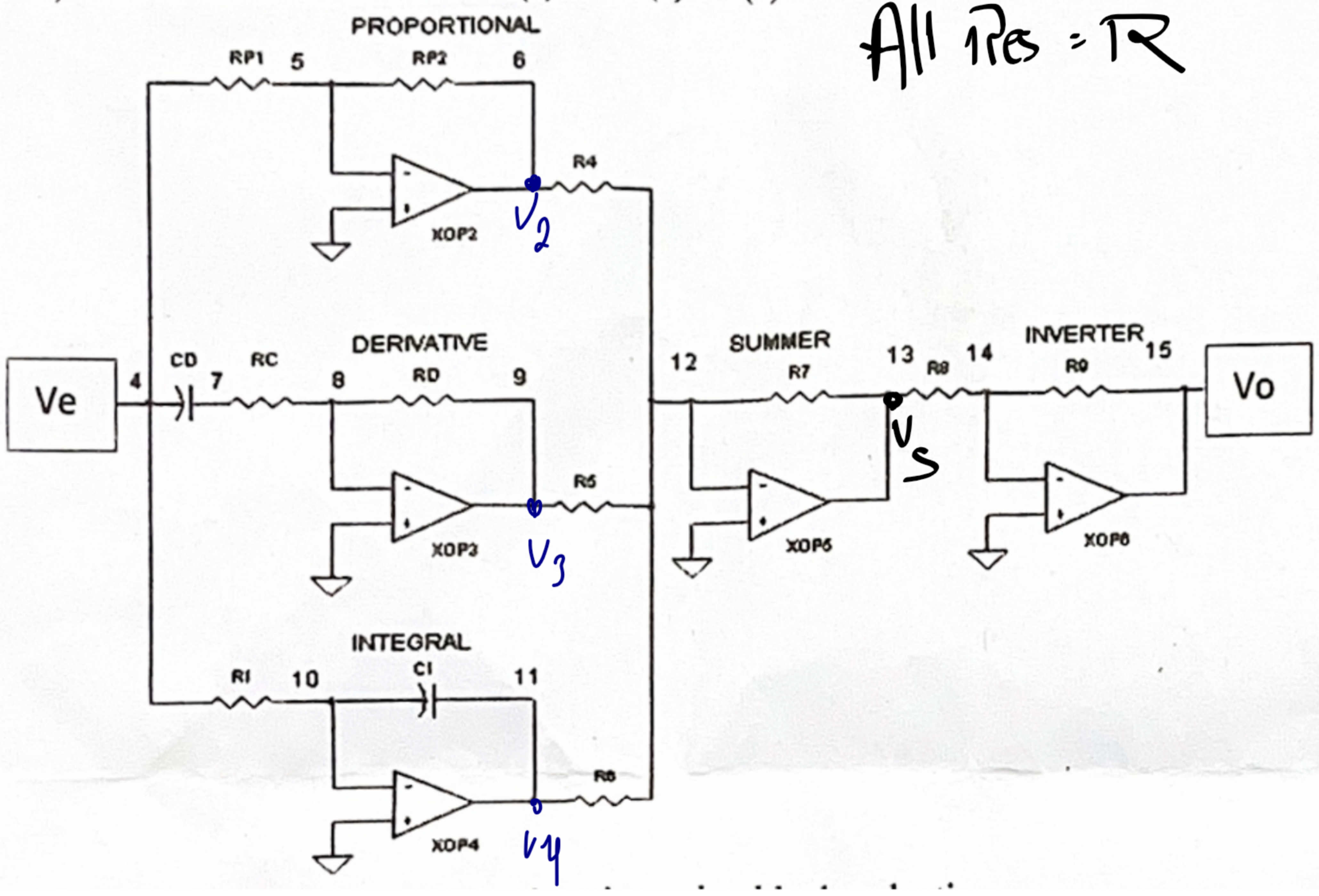
a) Find the transfer function $G(s) = Vo(s)/Ve(s)$

mid 59

All Res = R



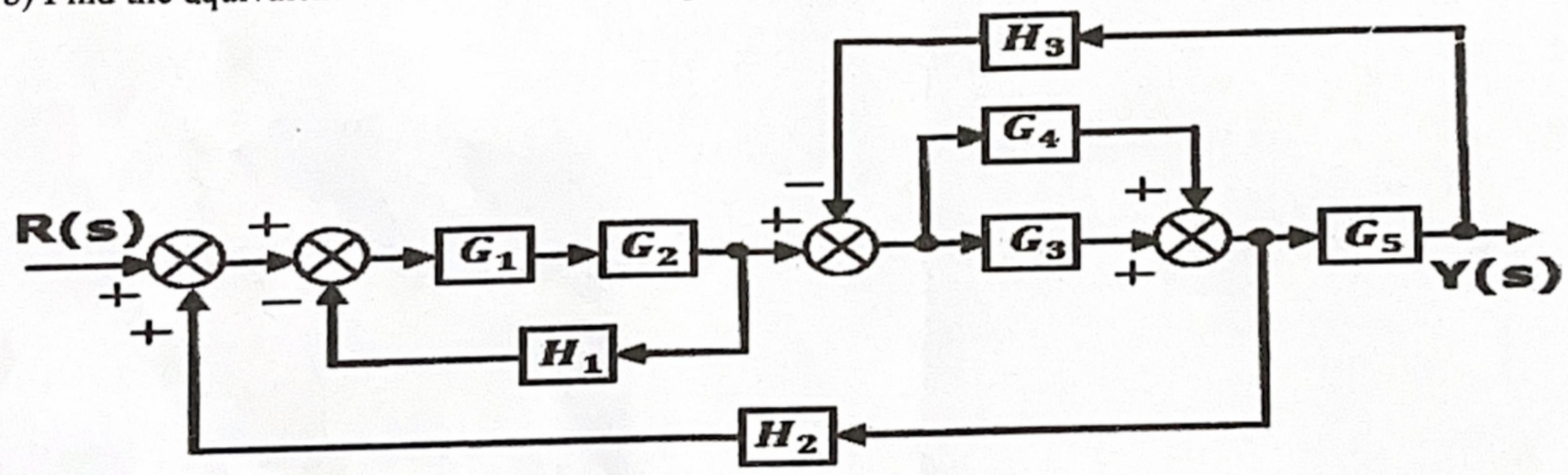
$$v_2 = -v_e \quad , \quad v_3 = \frac{-R}{R+\frac{1}{sC}} v_e \quad , \quad v_4 = -\frac{1}{sRC} v_e$$

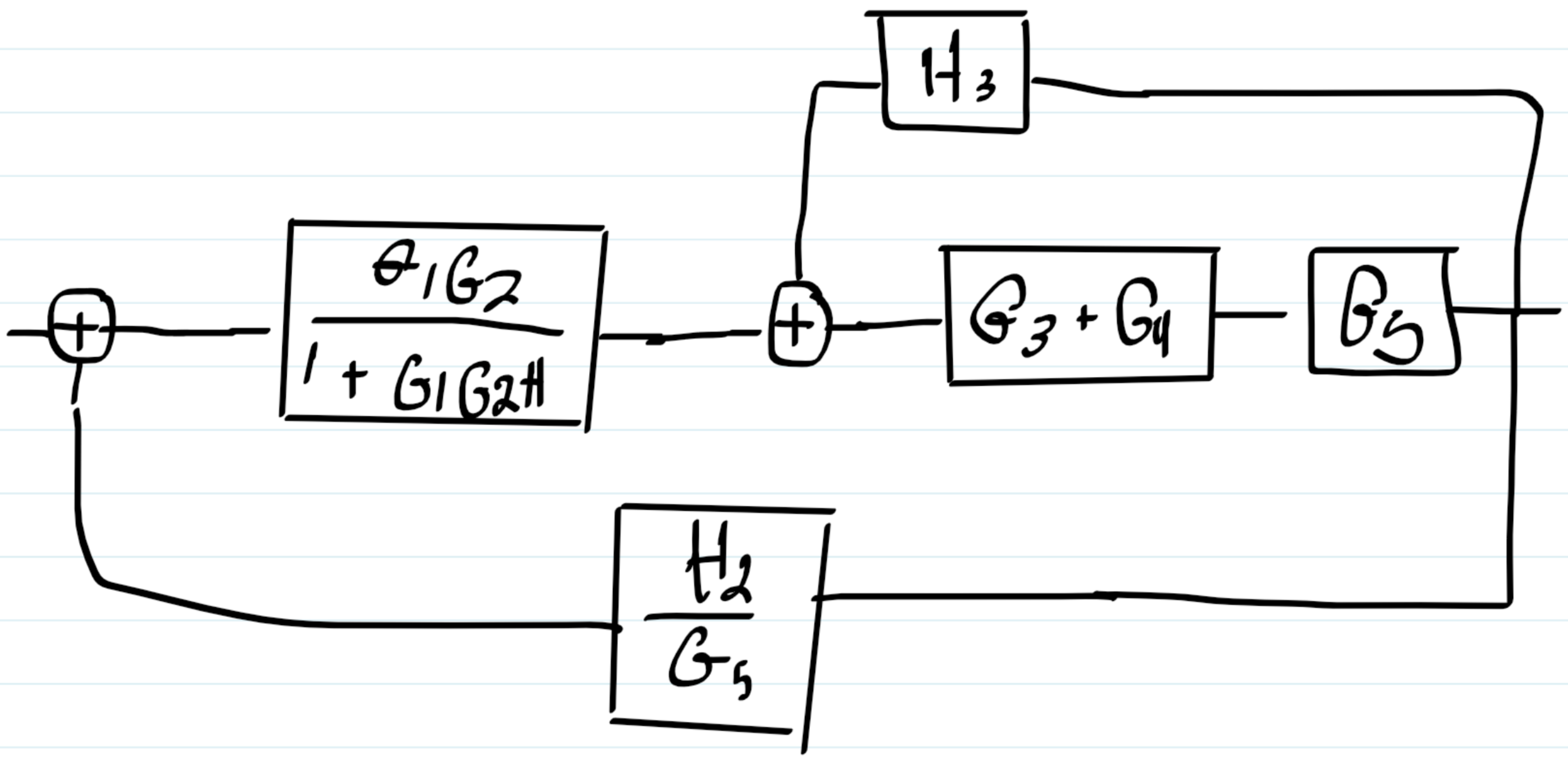
$$v_s = -\frac{\cancel{R_7}}{\cancel{R_4}} v_2 - \frac{\cancel{R_7}}{\cancel{R_5}} v_3 - \frac{\cancel{R_7}}{\cancel{R_6}} v_4$$

$$v_s = v_e + \frac{R}{R+\frac{1}{sC}} v_e + \frac{1}{sRC} v_e = V_e \left[1 + \frac{RSC}{RSC+1} + \frac{1}{RSC}\right]$$

$$v_0 = \frac{-R_9}{R_9} v_s \qquad \therefore G(s) = -1 - \frac{RSC}{RSC+1} - \frac{1}{RSC}$$

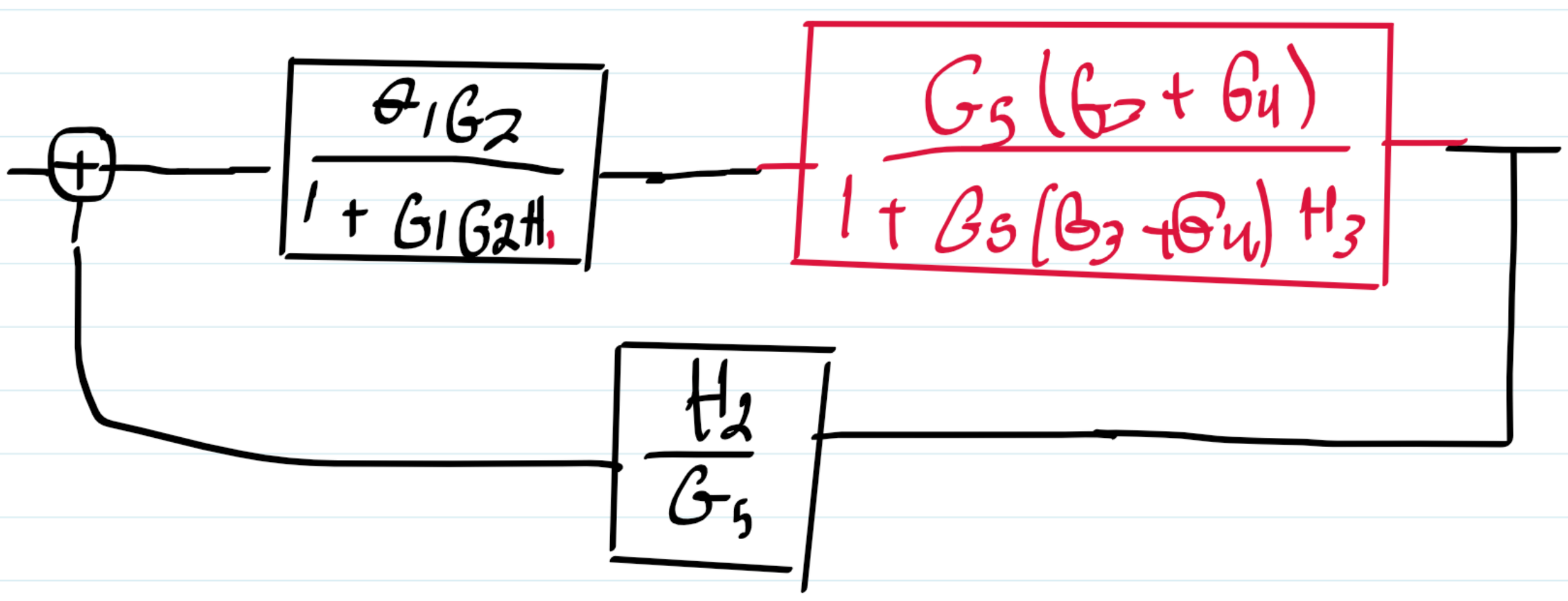
b) Find the equivalent transfer function using block reduction.



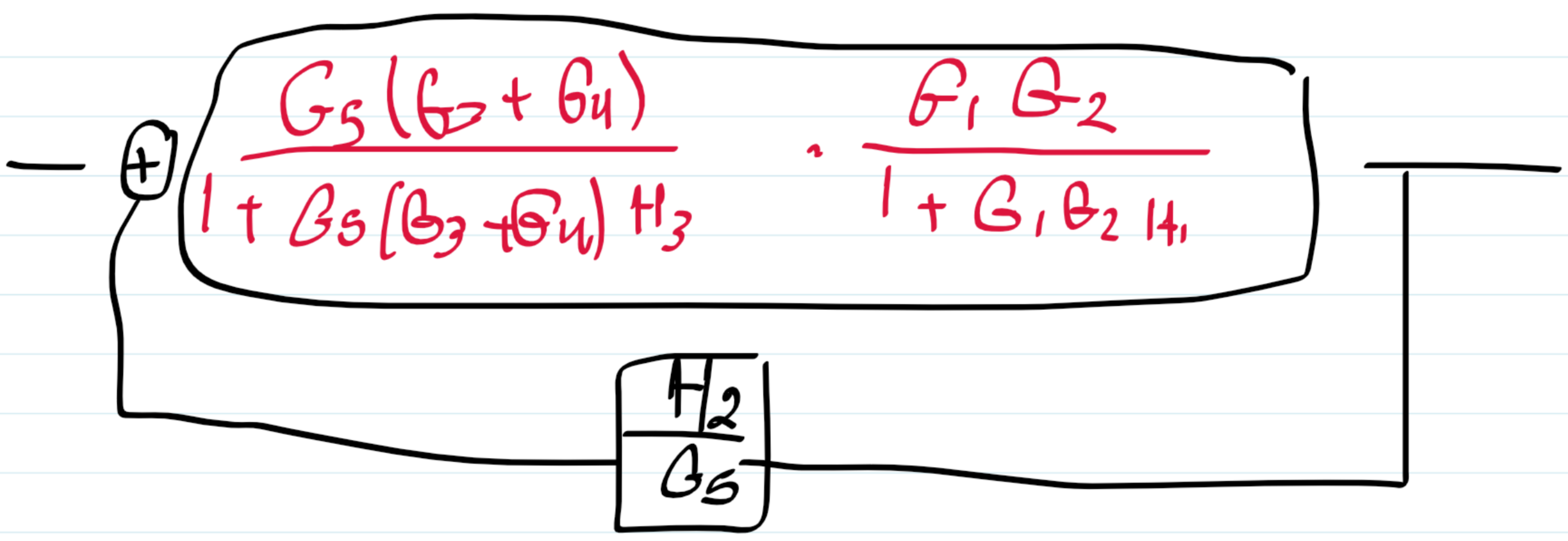


$$\frac{G_5(G_3+G_4)}{1+G_5(G_3+G_4)H_3}$$

=



$$\frac{G_5(G_3+G_4)}{1+G_5(G_3+G_4)H_3}\cdot\frac{G_1G_2}{1+G_1G_2H_1} =$$



$$\frac{\dfrac{G_1G_2G_5(G_3+G_4)}{1+G_1G_2H_1+G_5H_3(G_3+G_4)+G_1G_2G_5(G_3+G_4)H_1H_3}}{1-\dfrac{G_1G_2\cancel{G_5}(G_3+G_4)\cdot\dfrac{H_2}{\cancel{G_5}}}{1+G_1G_2H_1+G_5H_3(G_3+G_4)+G_1G_2G_5(G_3+G_4)H_1H_3}}$$

اضرب في المقام

Question 2: feedback control systems performance and characteristics (12 marks)

a) The transfer function of a closed-loop, unity feedback control system is: T(s) = K / (s^2 + 4s + K). If the system gain (K) is set to 16, calculate the rise time, maximum overshoot, and settling time (2%).

$$T(s) = \frac{16}{s^2 + 4s + 16}$$

$w_n^2 = 16 \therefore w_n = 4$

$2 w_n \zeta = 4 \quad \therefore \zeta = .5$

$w_d = w_n \sqrt{1-\zeta^2} = 2\sqrt{3} \qquad \theta = \tan^{-1} \frac{w_d}{\zeta w_n} = \frac{\pi}{3}$

$t_r = \frac{\pi - \theta}{w_d} = .604$

$m_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = .16 \quad , \quad t_{s\,2\%} = \frac{4}{\zeta w_n} = 2$

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b) For the following control system, find: G(s) = K(s+3) / (s²(s+4))

1. Kp, Kv, and Ka.
2. Using part 1, find Ess for unit step, ramp, and parabolic inputs respectively.

1) $K_p = \lim_{s \to 0} G(s) H(s) = \lim_{s \to 0} \frac{K(s+3)}{s^2(s+4)} = \infty$

$K_v = \lim_{s \to 0} s G(s) H(s) = \lim_{s \to 0} \frac{\cancel{s} K(s+3)}{s^{\cancel{2}}(s+4)} = \infty$

$K_a = \lim_{s \to 0} s^2 G(s) H(s) = \lim_{s \to 0} \frac{\cancel{s^2} K(s+3)}{\cancel{s^2}(s+4)} = \frac{3K}{4}$

2) $E_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$

$E_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0$

$E_{ss} = \frac{1}{K_a} = \frac{1}{\frac{3K}{4}} = \frac{4}{3K}$

a) A unity feedback control system has the open-loop transfer function: G(s) = K / (s(s+2)(s+5)). Determine the range of K for stability using the Routh-Hurwitz criterion

$$T(s) = \frac{\frac{K}{s(s+2)(s+5)}}{1 + \frac{K}{s(s+2)(s+5)}} = \frac{K}{s^3 + 7s^2 + 10s + K}$$

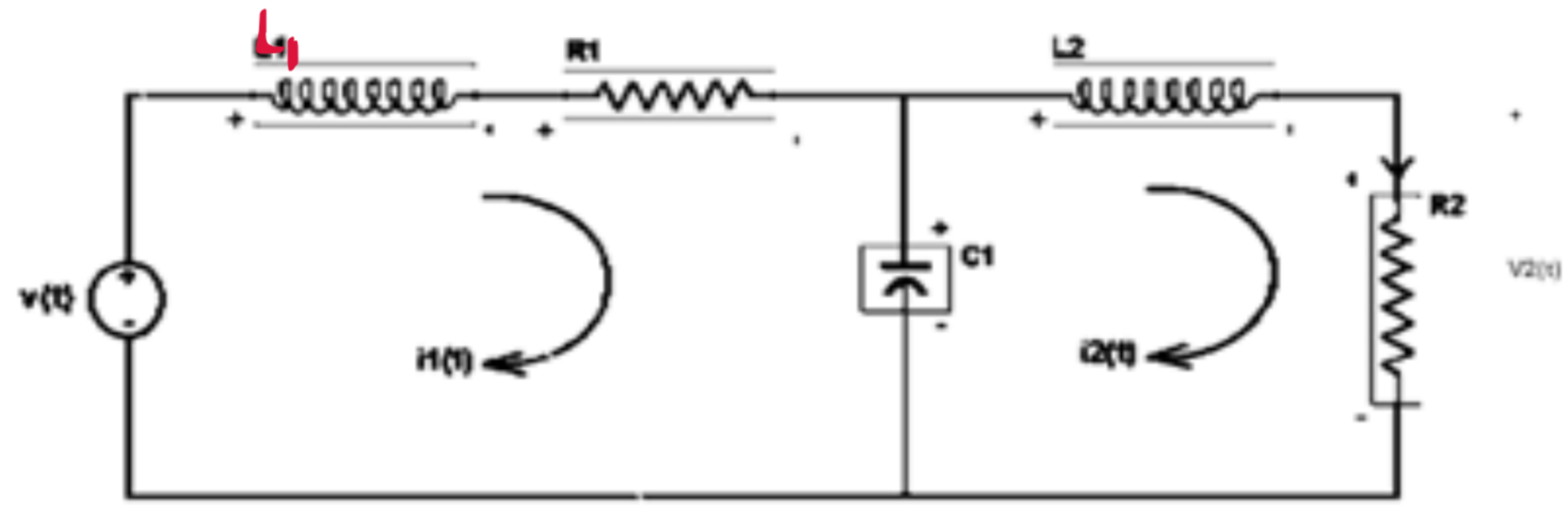
$$\begin{array}{llll} s^3 & 1 & & 10 \\ s^2 & 7 & & K \\ s^1 & \frac{-\begin{vmatrix} 1 & 10 \\ 7 & K \end{vmatrix}}{7} & = \frac{-(K-70)}{7} & 0 \\ s^0 & \frac{-\begin{vmatrix} 7 & K \\ \frac{-K+70}{7} & 0 \end{vmatrix}}{\frac{-K+70}{7}} & = K & 0 \end{array}$$

$\therefore \quad \frac{-K+70}{7} > 0 \quad \therefore \quad K < 70$

$\therefore \quad \boxed{0 < K < 70}$

mid 58

a) find the transfer function $G(s) = i_2(s)/v(s)$



$$v = i_1\left[sL_1 + R_1 + \frac{1}{sC_1}\right] - i_2\frac{1}{sC_1} \qquad (1)$$

$$0 = i_2\left[sL_2 + R_2 + \frac{1}{sC_1}\right] - i_1\frac{1}{sC_1}$$

$$\therefore\ i_1 = i_2\, sC_1\left[sL_2 + R_2 + \frac{1}{sC_1}\right]$$

sub in (1)

$$v = i_2\, sC_1\left[sL_2 + R_2 + \frac{1}{sC_1}\right]\left[sL_1 + R_1 + \frac{1}{sC_1}\right] - i_2\frac{1}{sC_1}$$

$$= i_2\left[sC_1\left[sL_2 + R_2 + \frac{1}{sC_1}\right]\left[sL_1 + R_1 + \frac{1}{sC_1}\right] - \frac{1}{sC_1}\right]$$

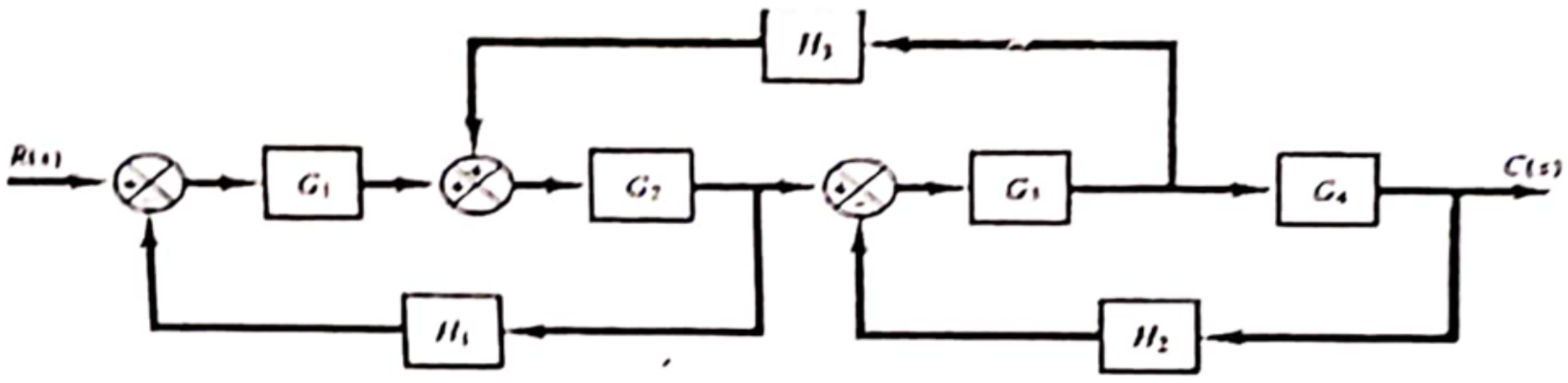
$$\therefore\ \frac{i}{v} = \frac{1}{sC_1\left[sL_2 + R_2 + \frac{1}{sC_1}\right]\left[sL_1 + R_1 + \frac{1}{sC_1}\right] - \frac{1}{sC_1}}$$

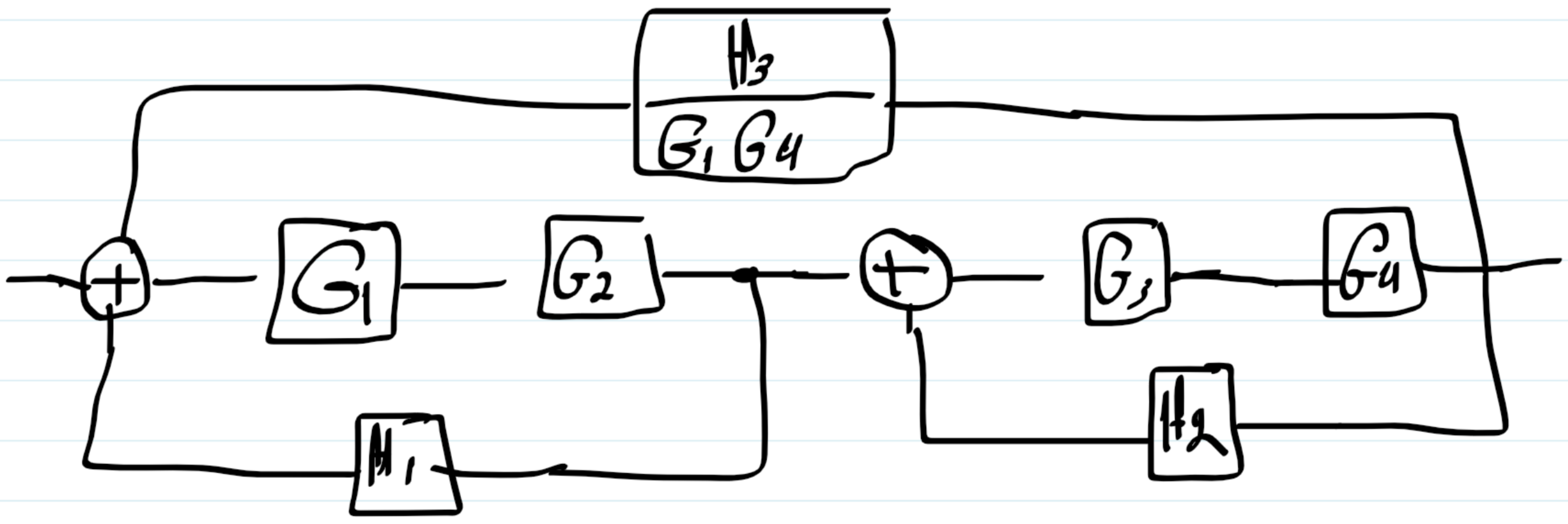
$$\frac{1}{\left[s^2C_1L_2 + R_2 sC_1 + 1\right]\left[sL_1 + R_1 + \frac{1}{sC_1}\right] - \frac{1}{sC_1}} =$$

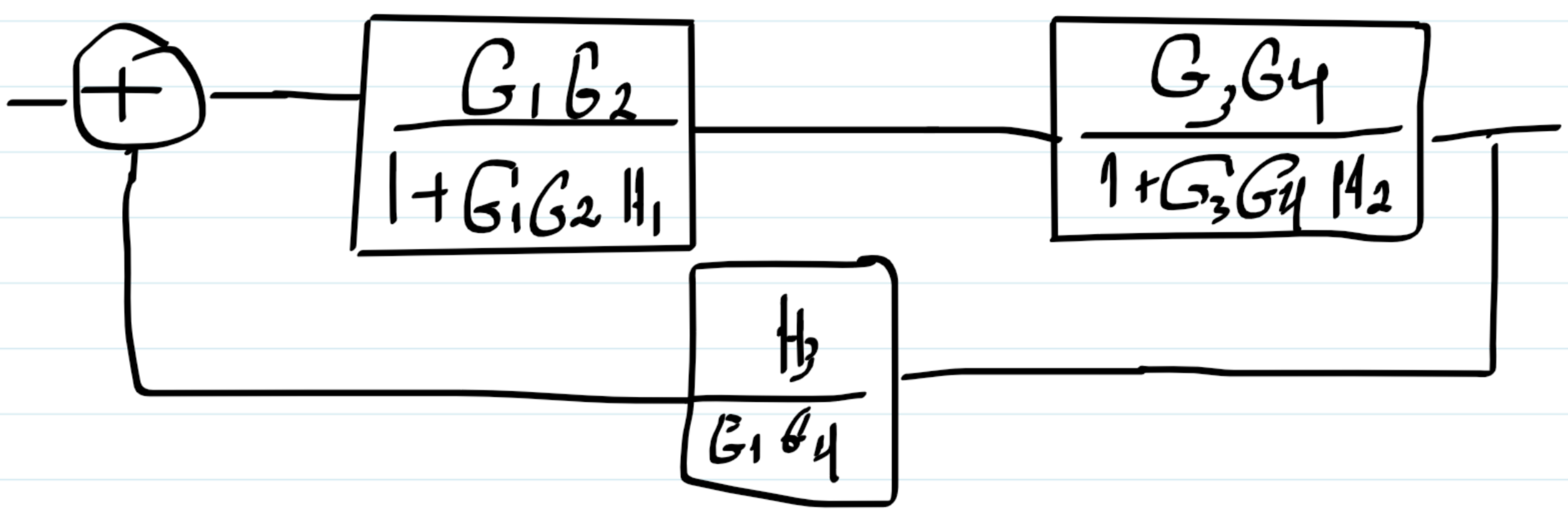
$$\frac{1}{s^3C_1L_1L_2 + s^2C_1L_2R_1 + sL_2 + R_2s^2C_1L_1 + R_1R_2sC_1 + R_2 + sL_1 + R_1 + \cancel{\frac{1}{sC_1}} - \cancel{\frac{1}{sC_1}}}$$

$$= \frac{1}{s^3(C_1L_1L_2) + s^2(C_1L_2R_1 + R_2C_1L_1) + s(L_2 + R_1R_2C_1 + L_1) + (R_2 + R_1)}$$

b) find the equivalent transfer function using block reduction.







$$\Rightarrow \quad \frac{G_1 G_2 G_3 G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}$$

$$= \frac{\dfrac{G_1 G_2 G_3 G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}}{1 + \dfrac{\cancel{G_1} G_2 G_3 \cancel{G_4}}{(1+G_1G_2H_1)(1+G_3G_4H_2)} \cdot \dfrac{H_3}{\cancel{G_1G_4}}} = \frac{G_1 G_2 G_3 G_4}{1+G_3G_4H_2+G_1G_2H_1+G_1G_2G_3G_4H_1H_2+G_2G_3H_3}$$

a) The T.F. of a closed-loop, unity feedback control system is: $\frac{C(S)}{R(S)} = \frac{K}{S^2 + 2S + K}$

If the system gain (K) is set to 10 Calculate the rise time, maximum overshoot, and settling time (2%)

$$TF = \frac{10}{s^2 + 2s + 10}$$

$\therefore \omega_n = \sqrt{10}$ $\quad 2\xi\omega_n = 2$ $\quad \therefore \xi = \frac{1}{\sqrt{10}}$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 3$ $\quad \theta = \cos^{-1}\xi = 1.25$

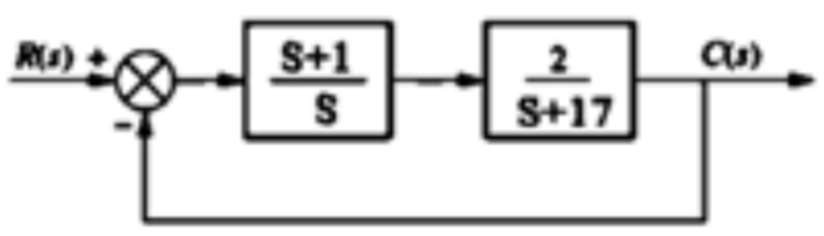
$\therefore t_r = \frac{\pi-\theta}{\omega_d} = .63$

$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = .3509$

$t_{s\,2\%} = \frac{4}{\xi\omega_n} = 4$

b) for the following control system find:

1. Kp, Kv, Ka,
2. Using part 1 find E_{ss} for unit step, ramp and parabolic inputs respectively



$G(s) = \frac{2(s+1)}{s(s+17)}$ $\quad H(s) = 1$

$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{2(s+1)}{s(s+17)} = \infty$

$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{2(s+1)\cancel{s}}{\cancel{s}(s+17)} = \frac{2}{17}$

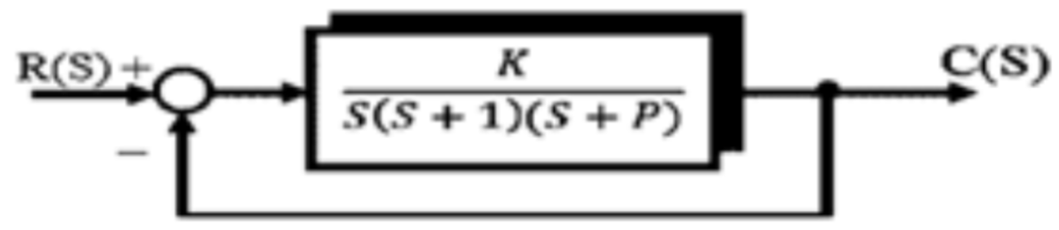
$K_a = \lim_{s\to 0} s^2G(s)H(s) = \lim_{s\to 0} \frac{2(s+1)s}{\cancel{s}(s+17)} = 0$

$E_{ss} = \frac{1}{1+K_p} = 0$

$E_{ss} = \frac{1}{K_v} = \frac{17}{2}$

$E_{ss} = \frac{1}{K_a} = \infty$

c) It is found that, the unity feedback control system shown in Fig. 3, is stable for the range $0 <= K < 2$
Determine the value of P to fulfill this condition.



$$T(s) = \frac{K}{s^3 + (1+P)s^2 + Ps + K}$$

$$\begin{array}{lll}
s^3 & 1 & P \\
s^2 & 1+P & K \\
s^1 & \dfrac{-\begin{vmatrix} 1 & P \\ 1+P & K \end{vmatrix}}{1+P} = \dfrac{-K+P+P^2}{1+P} & 0 \\
s^0 & \dfrac{-\begin{vmatrix} 1+P & K \\ \frac{-K+P+P^2}{1+P} & 0 \end{vmatrix}}{\frac{-K+P+P^2}{1+P}} = K & 0
\end{array}$$

$$\therefore \frac{-K+P+P^2}{1+P} > 0$$

$$K < P+P^2 \quad \& \quad K < 2$$

$$\therefore P+P^2 = 2$$

$$\therefore P = 1, -2$$

$$\because 1+P > 0$$

$$\boxed{\therefore P = 1}$$

Thought for 23s

P = 1

Why: For unity feedback with $G(s) = \frac{K}{s(s+1)(s+P)}$, the closed-loop characteristic polynomial is

$$s(s+1)(s+P) + K = s^3 + (1+P)s^2 + Ps + K = 0.$$

Routh array (cubic) first column:

$$\begin{array}{c|cc}
s^3 & 1 & P \\
s^2 & 1+P & K \\
s^1 & \dfrac{P(1+P)-K}{1+P} & 0 \\
s^0 & K &
\end{array}$$

Stability requires all first-column terms > 0:

- $1 > 0$ (ok),
- $1+P > 0 \Rightarrow P > -1$,
- $K > 0$,
- $\frac{P(1+P)-K}{1+P} > 0 \Rightarrow K < P(1+P)$ (since $1+P > 0$).

Thus the allowable gain range is $0 < K < P(1+P)$. Given the system is stable for $0 \le K < 2$, we need

$$P(1+P) = 2 \;\Rightarrow\; P^2 + P - 2 = 0 \;\Rightarrow\; P = 1 \text{ or } P = -2.$$

But $P > -1$, so the valid value is $P = 1$.