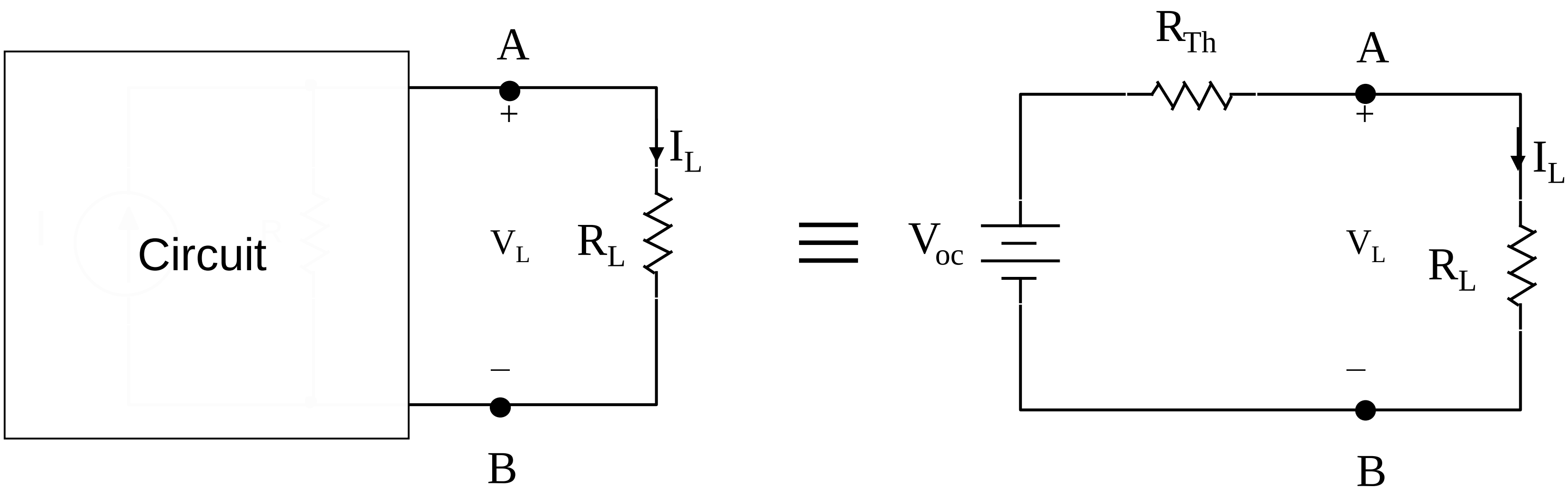
Electric Circuits I
ELEC 305

Thevenin's and Norton's Theorems

Thevenin's Theorem



V_{oc} : Open-circuit voltage between terminals A & B

R_{Th}: Equivalent resistance between terminals A &B with all independent sources are killed

Applying Thevenin's Theorem

- Remove the load (open the circuit between the load terminals)
- Find V_{oc} by applying all known circuit analysis techniques
- Determine the equivalent resistance of the circuit at the terminals (R_{Th})
- Connect the load to V_{oc} and R_{Th} to the load terminals and determine the required current or voltage .

Thevenin's Equivalent Resistance

The calculation of R_{Th} depends on the type of sources in the circuit as follows:

1- Independent Sources only:

voltage and current sources are killed and the equivalent resistance is calculated

2- Dependent Sources only:

an external independent voltage /current source is applied at the open terminals and the corresponding current/voltage is determined. The voltage/current ratio is Thevenin's resistance (R_{Th})

Thevenin's Equivalent Resistance

3- Both Independent and Dependent Sources:

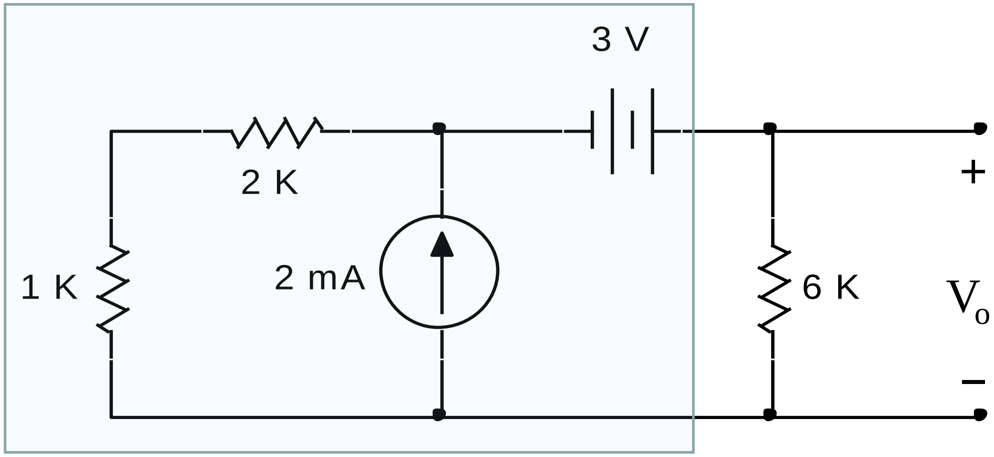
The open terminals are shorted and the short-circuit current is determined. The ratio of the open-circuit voltage to the short-circuit current is Thevenin's resistance (R_{Th})

1- Circuits Containing Only Independent Sources

Example #1

Find the voltage V_o using

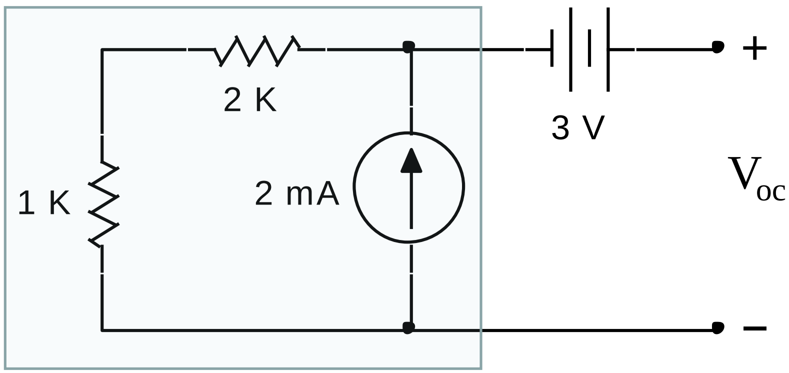
(a) Thevenin's theorem



Solution using Thevenin's Theorem

1- Open the circuit at the load terminals

2- Calculate V_{oc}



Using source transformation, we get a 6 V source

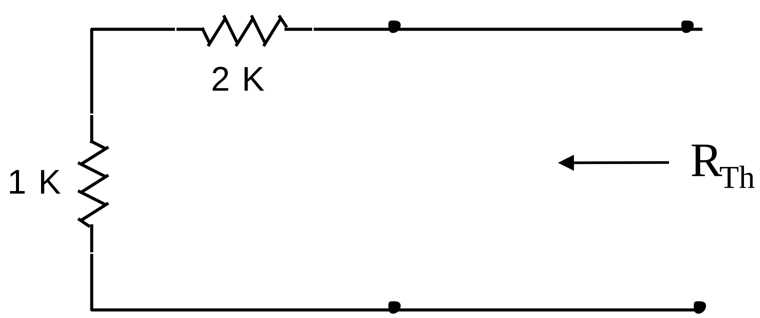
6 V source in series with a resistor and the 3 V source →

$$V_{oc} = 3 + 6 = 9\ V$$

Solution using Thevenin's Theorem

3- Determine R_{Th}:

– Short-circuit the voltage sources and open-circuit the current source →

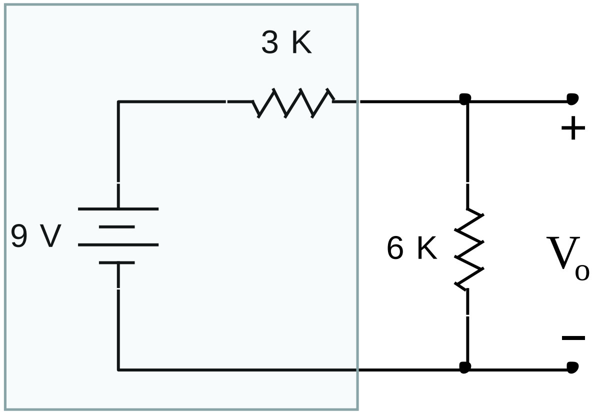


– Calculate the equivalent resistance

$$R_{Th} = 2\ K + 1\ K = 3\ K$$

Solution using Thevenin's Theorem

4- Connect the load to Thevenin's equivalent circuit, and calculate V_o

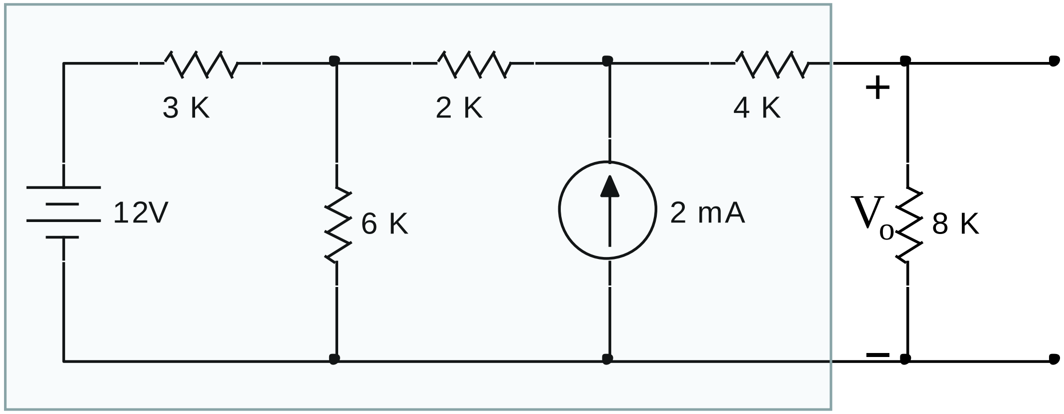


Using voltage division rule:

$$V_o = 9*6 / (6+3) = 6\ V$$

Example #2

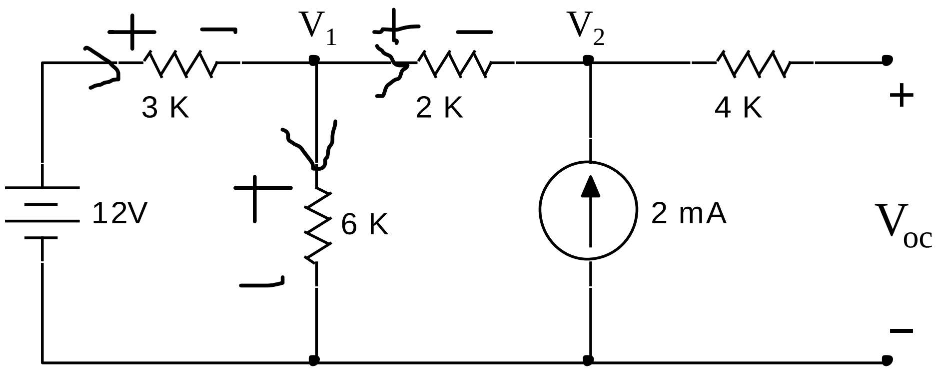
Find the voltage V_o using Thevenin's theorem.



1- Open the circuit at the load terminals

2- Calculate V_{oc}

Solution



Using nodal analysis:

$(12 - V_1) / 3 \; - \; V_1/6 \; - \; (V_1 - V_2) / 2 \; = \; 0 \qquad (1)$

$(V_1 - V_2) / 2 \; + \; 2 \; = \; 0 \qquad (2)$

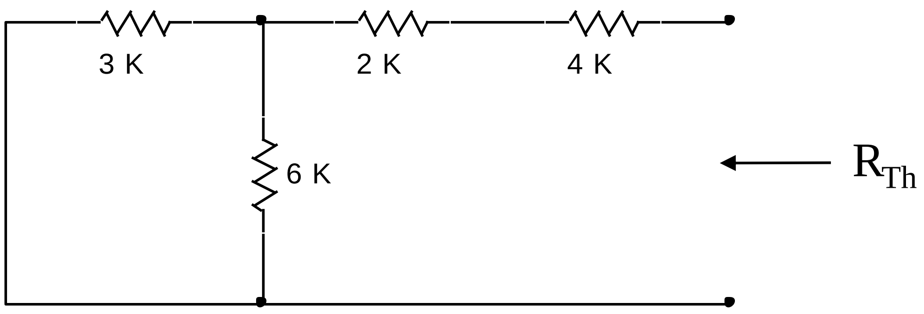
Solving the equations simultaneously yields:

$V_2 = 16 \quad \rightarrow \quad V_{oc} = 16$

Solution

3- Determine R_{Th}:

- Short-circuit the voltage sources and open-circuit the current source →

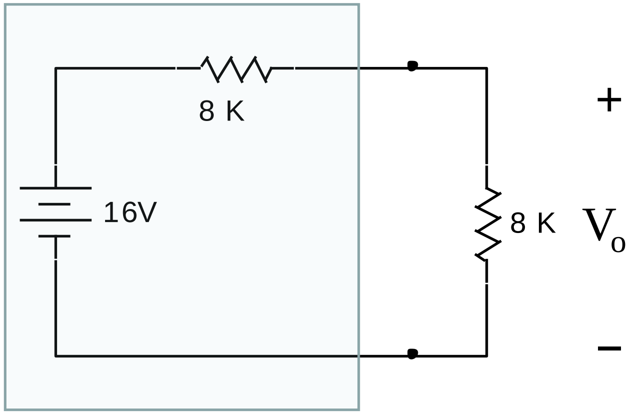


- Calculate the equivalent resistance

$$R_{Th} = 2\text{ K} + 4\text{ K} + (6\text{ K} \,//\, 3\text{ K}) = 8\text{ K}$$

Solution

4- Connect the load to Thevenin's equivalent circuit, and calculate V_o

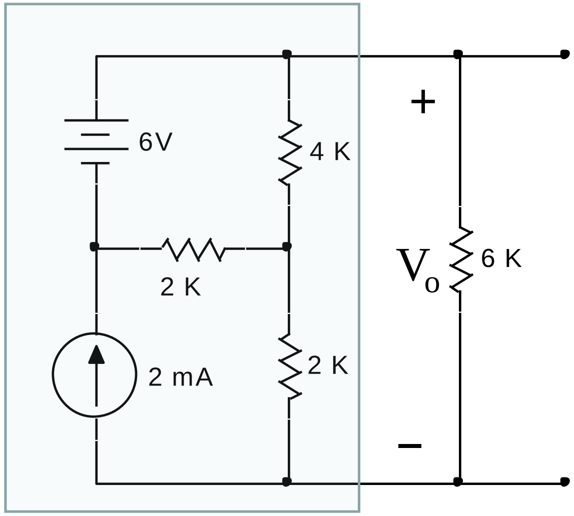


Using simple voltage division:

$$V_o = 16 / 2 = 8 \text{ V}$$

Example #3

Find the voltage V_o using Thevenin's theorem



1- Open the circuit at the load terminals

2- Calculate V_{oc}

Solution

Using loop analysis:

$I_2 = 2\text{ mA}$

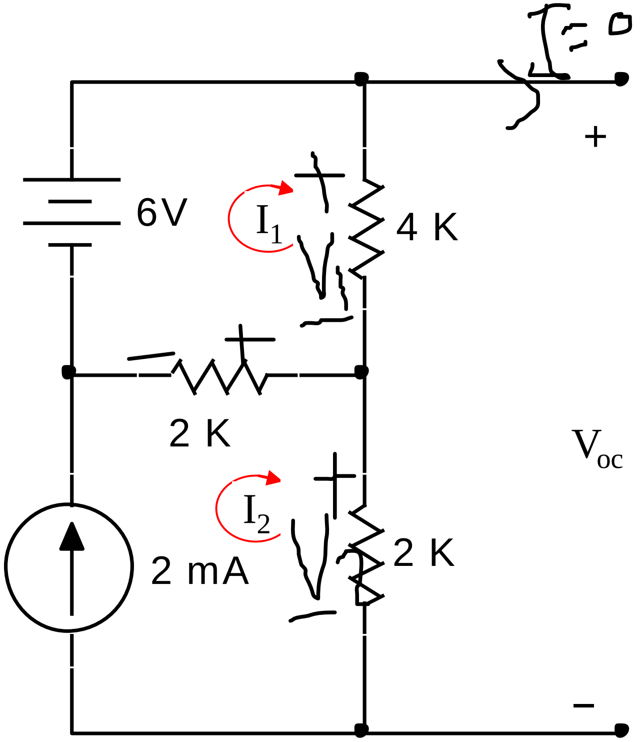
$4\ I_1 + 2(I_1 - I_2) - 6 = 0$

$\rightarrow$

$I_1 = 5/3\text{ mA}$

$V_{oc} = 4\ I_1 + 2\ I_2$

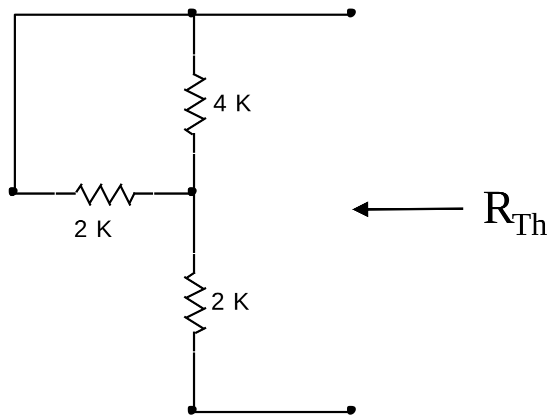
$= 4*5/3 + 2*2 = 32/2$



Solution

3- Determine R_{Th}:

– Short-circuit the voltage sources and open-circuit the current source →

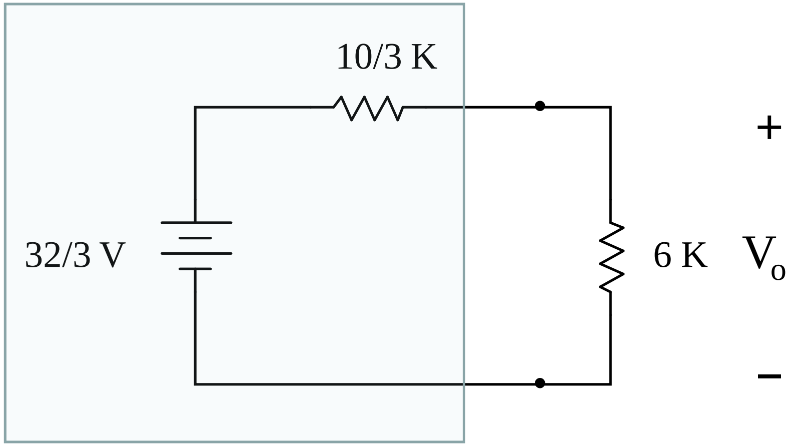


– Calculate the equivalent resistance

$$R_{Th} = 2\ K + (4\ K\ //\ 2\ K) = 10/3\ K$$

Solution

4- Connect the load to Thevenin's equivalent circuit, and calculate V_o

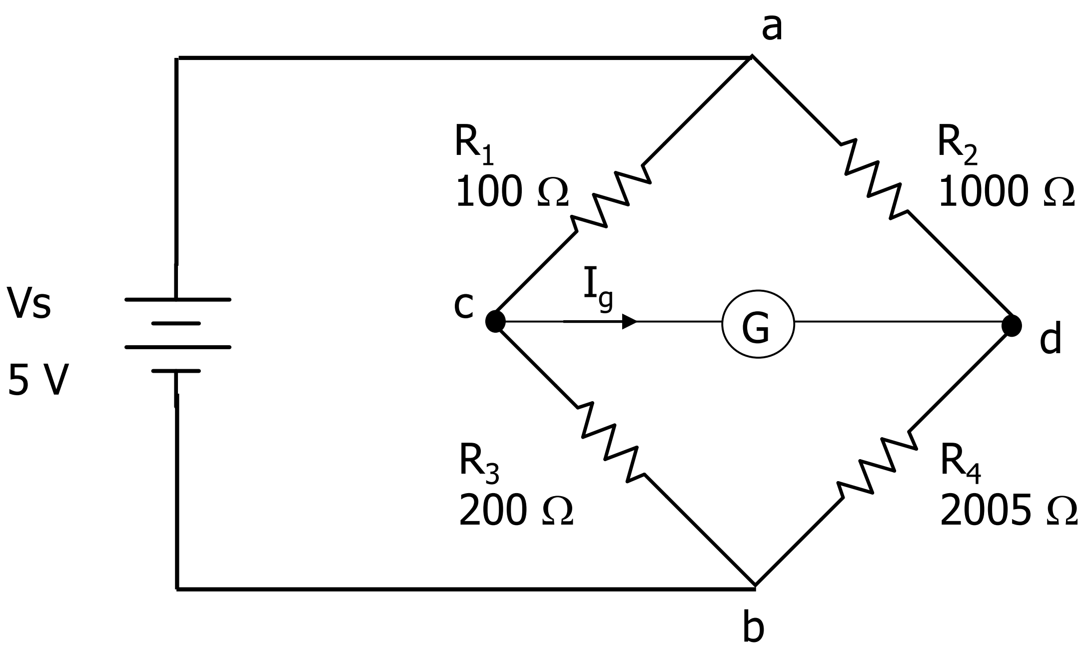


Using voltage division rule:

$$V_o = (32/2)*6 \,/\, (6+10/3) = 48/7 \text{ V}$$

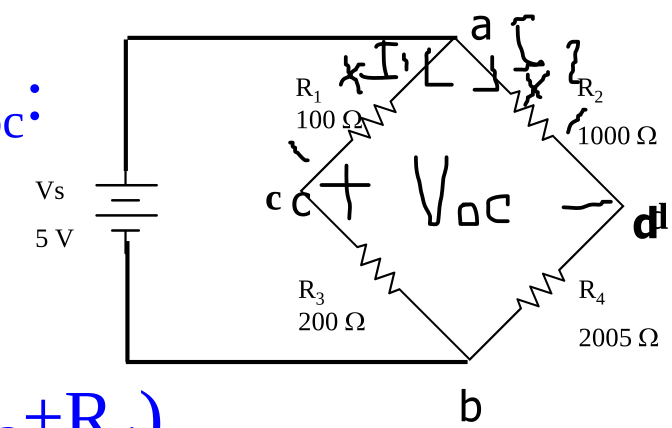
Example #4: Wheatstone Bridge

Calculate the current I_g, if the resistance of the galvanometer is 100 Ω.



Solution

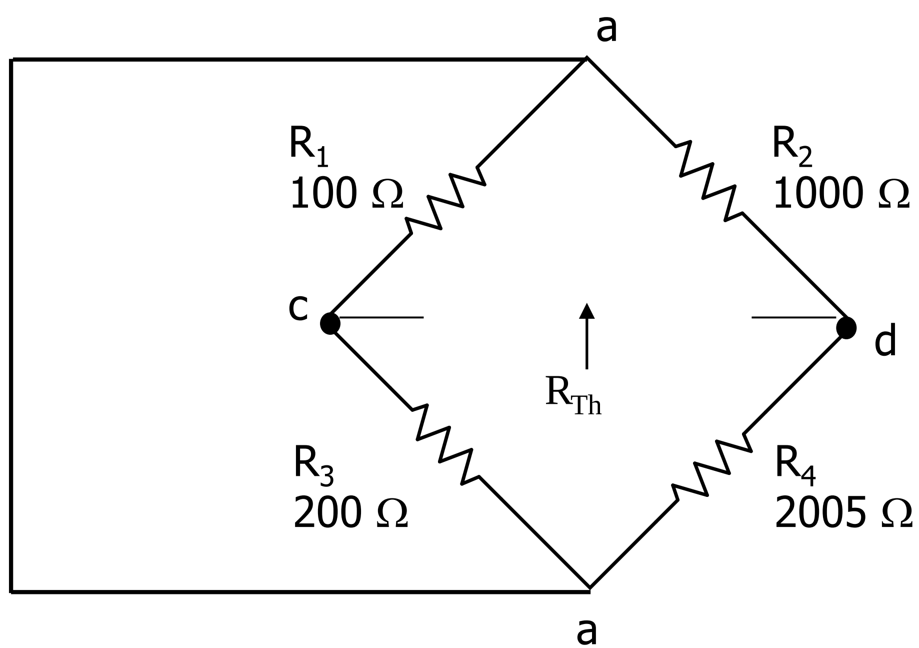
- Open the circuit between terminals ‘c’ and ‘d’, and find the equivalent voltage and equivalent resistance



- Thevenin’s equivalent voltage V_{oc}:

$$V_{oc} = V_{cb} - V_{db}$$
$$= I_1 R_3 - I_2 R_4$$
$$= V_s R_3/(R_1+R_3) - V_s R_4/(R_2+R_4)$$
$$= 5\,(200/300 - 2005/3005)$$
$$= -2.77 \text{ mV}$$

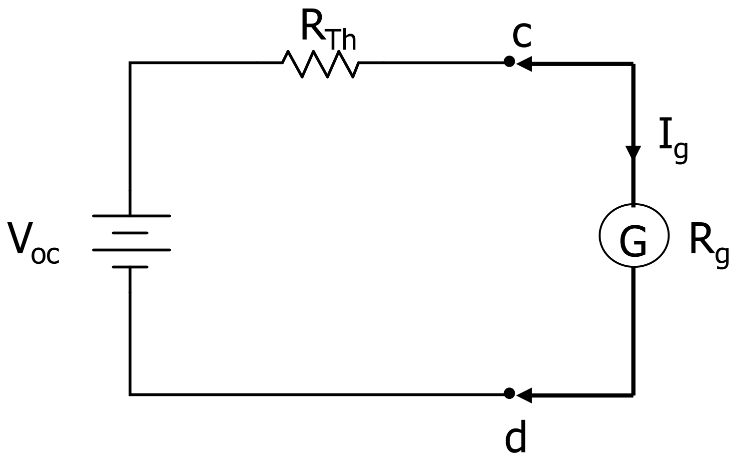
Solution



$$R_{Th} = R_1//R_3 + R_2//R_4$$

$$= 100//200 + 1000//2005 = 734\ \Omega$$

Solution



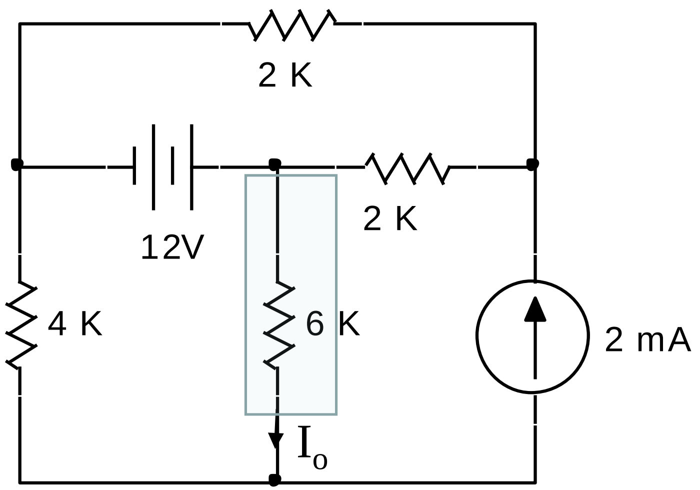
When the galvanometer is connected to the output terminals of the Thevenin's equivalent circuit:

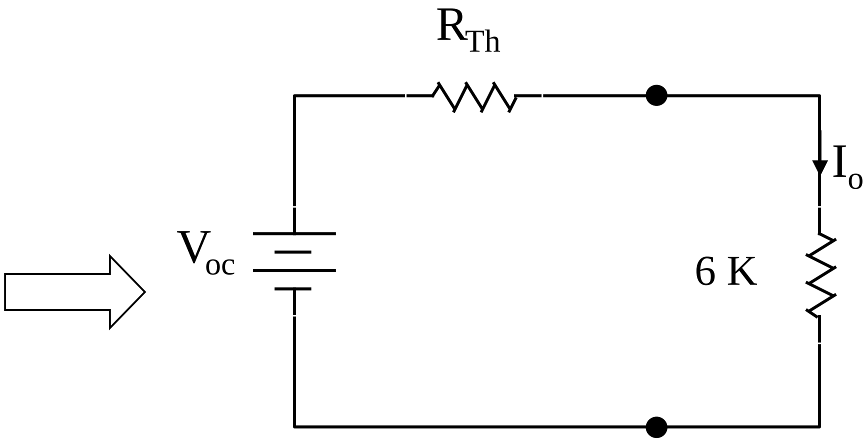
$$I_g = V_{oc} / (R_{Th} + R_g)$$

$$= -2.77 \text{ mV} / (734+100) = -3.4 \ \mu\text{A}$$

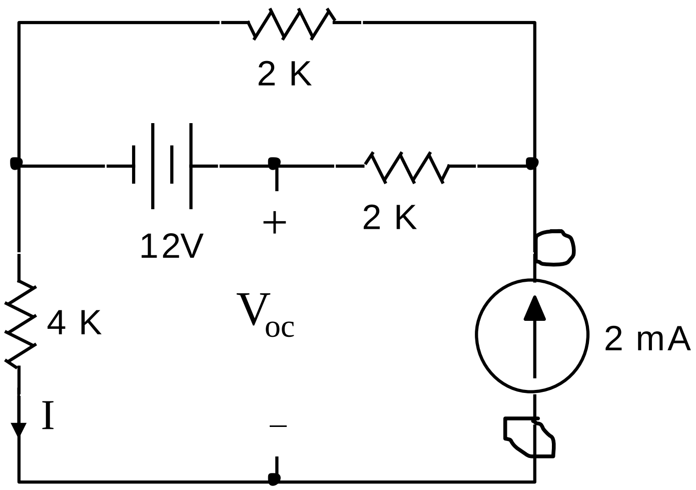
Example #5

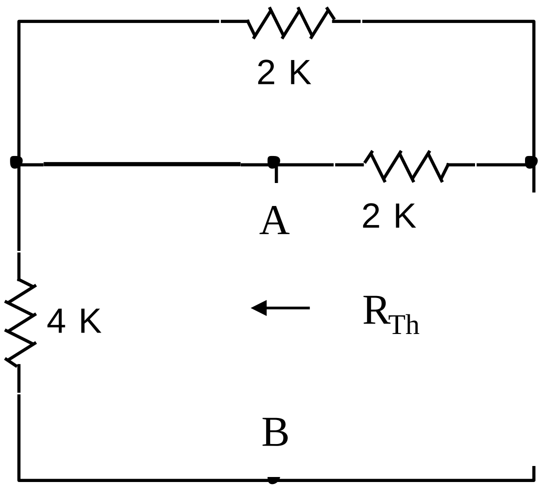
Use Thevenin's theorem to find I_o





Solution





$I = 2\ mA$

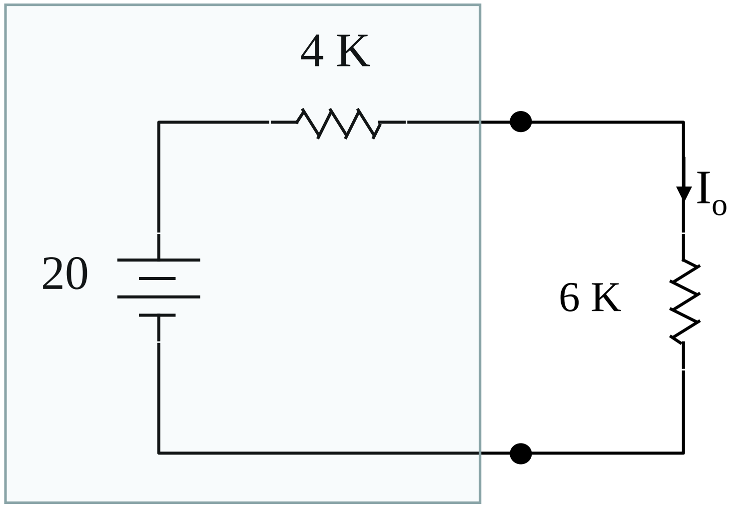
$V_{oc} = 12 + 4*I$

$= 12 + 4*2 = 20\ V$

Since the (2K+2K) is in parallel with a short circuit:

$R_{Th} = R_{AB} = 4\ K$

Solution



Connecting the load to Thevenin's equivalent circuit

$\rightarrow$ $I_o = 20 / (4+6)K$

$= 20 / 10\ K = 2\ mA$

Circuits Containing Only Dependent Sources

Circuits containing only dependent sources

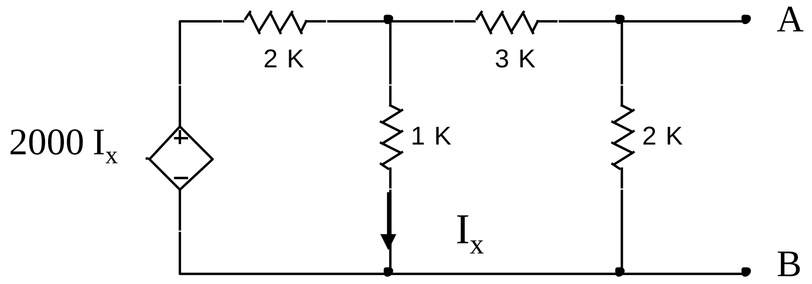
- In this case:
  - $V_{oc} = 0$
  - $I_{sc} = 0$

→Thevenin's / Norton's equivalent circuit consists of R_{Th} only

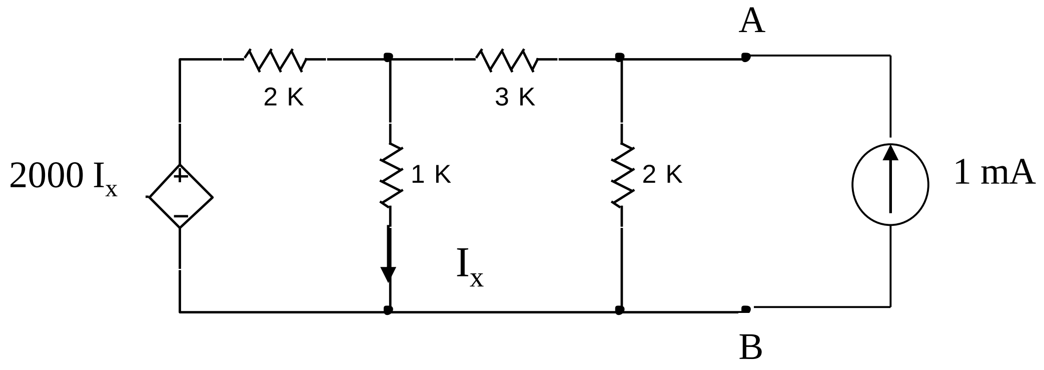
- To calculate R_{Th}, an independent voltage /current source is applied at the open terminals and the corresponding current/voltage is determined. The ratio between voltage and current is Thevenin's resistance (R_{Th})

Example #8

Determine Thevenin's equivalent circuit at terminals A-B.

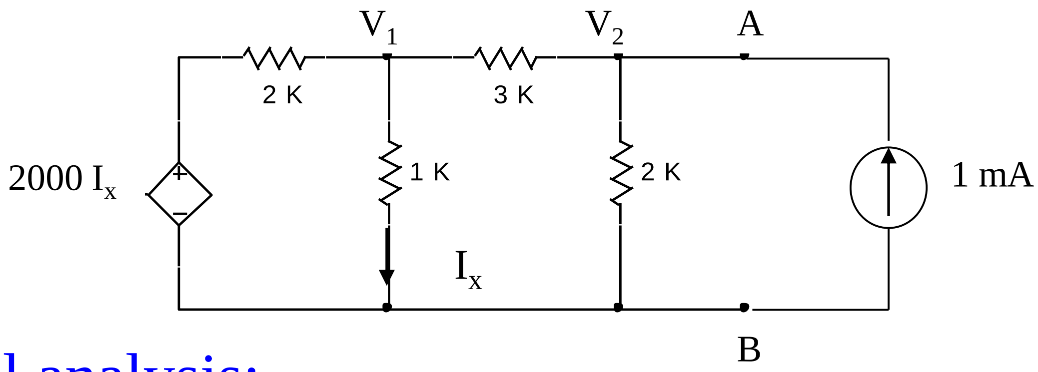


Solution: 1- Apply a current source at the terminals



Solution

2- Determine V_{AB}



Using nodal analysis:

$$(2\,I_x - V_1)/2 - V_1/1 - (V_1 - V_2)/3 = 0 \qquad (1)$$

$$(V_1 - V_2)/3 - V_2/2 + 1 = 0 \qquad (2)$$

Also,

$$I_x = V_1/1 \qquad (3)$$

Solution

Solving the equations simultaneously yields:

$$V_2 = 10/7$$

$$\rightarrow$$

$$V_{AB} = 10/7$$

3- Determine R_{Th}

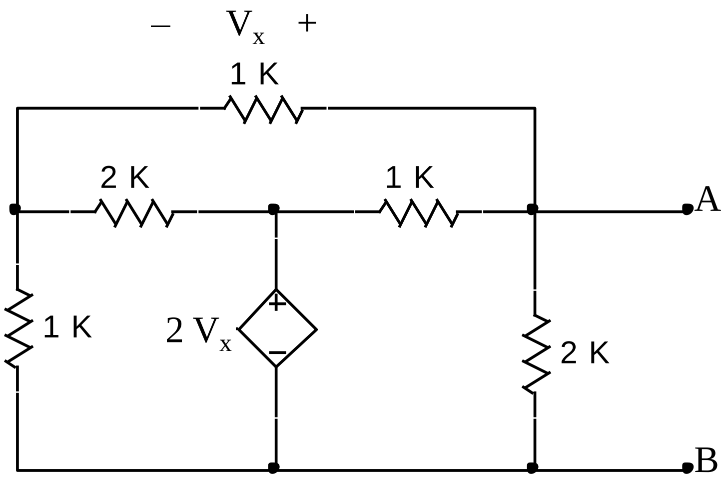
$$R_{Th} = V_{AB} / \text{(applied current)}$$

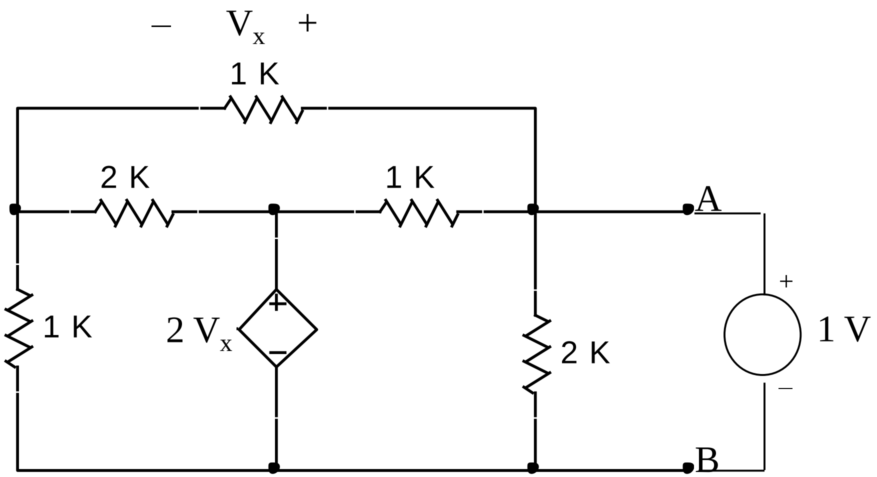
$$= V_{AB} / 1 \text{ mA}$$

$$= 10/7 \text{ K}$$

Example #9

Determine Thevenin's equivalent circuit at terminals A-B.

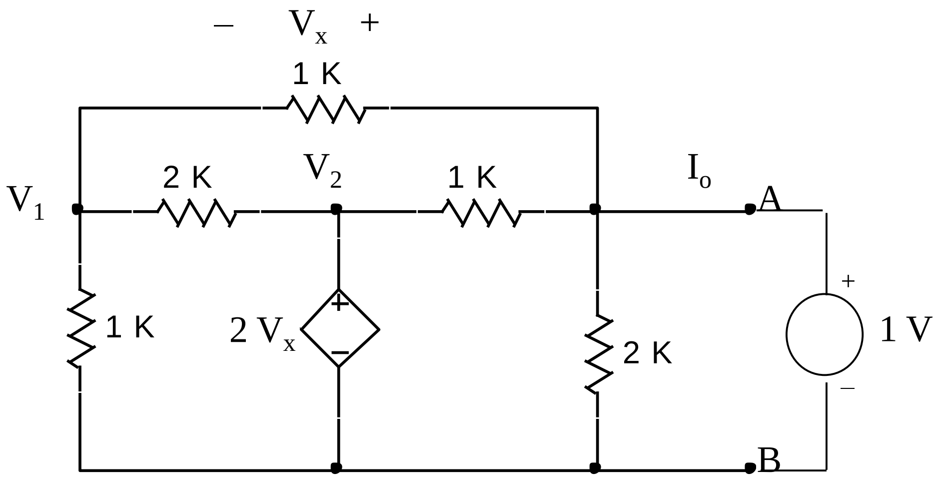




Solution: 1- Apply a voltage source at the terminals

Solution

2- Determine I_o



Using nodal analysis:

$$V_1/1 + (V_1 - V_2)/2 + (V_1 - 1)/1 = 0 \qquad (1)$$

$$V_2 = 2\,V_x \qquad (2)$$

Solution

Also,

$$1 - V_1 = V_x$$

Solving the equations simultaneously yields:

$$V_x = 3/7 \text{ V}$$

Hence the current I_o:

$$I_o = 1/2K + V_x/1K + (1 - 2V_x)/1K = 15/14 \text{ mA}$$

Finally,

$$R_{Th} = 1V / I_o = 14/15 \text{ K}$$

Circuits Containing Both Independent and Dependent Sources

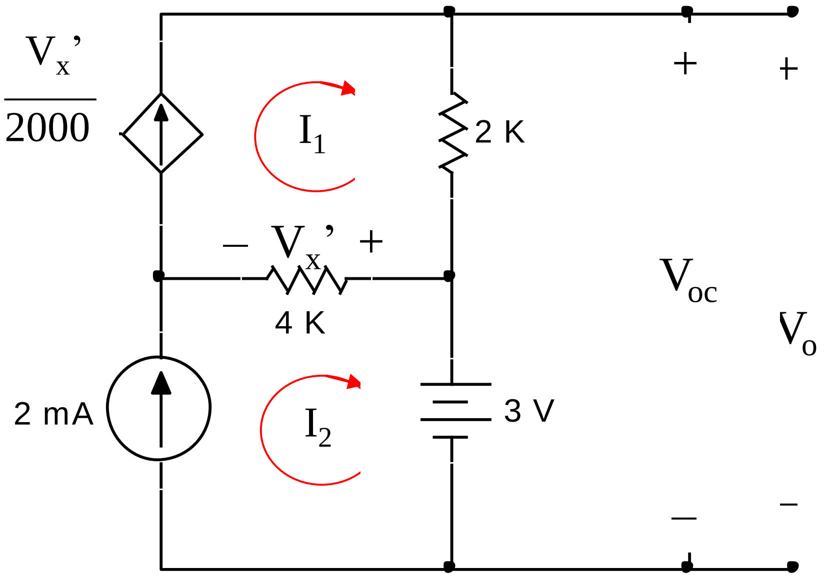
Circuits containing both independent and dependent sources

- In this case, both the open-circuit voltage (V_{oc}) and the short-circuit current (I_{sc}) are determined

- The equivalent resistance (R_{Th}) is calculated as:

$$R_{Th} = V_{oc} / I_{sc}$$

Use Thevenin's theorem to find V_o



Solution: First find V_{oc} using loop analysis:

$I_2 = 2\ \text{mA}$

$I_1 = V_x / 2\ \text{mA}$

$V_x = 4\,(I_1 - I_2)$

$I_1 = 4\,(I_1 - I_2) / 2$

→ $I_1 = 4\ \text{mA}$

$V_{oc} = 3 + 2\,I_1 = 11\ \text{V}$

Solution

Determine I_{sc} using nodal analysis, assuming the reference node as indicated in the circuit diagram.

At node 1:

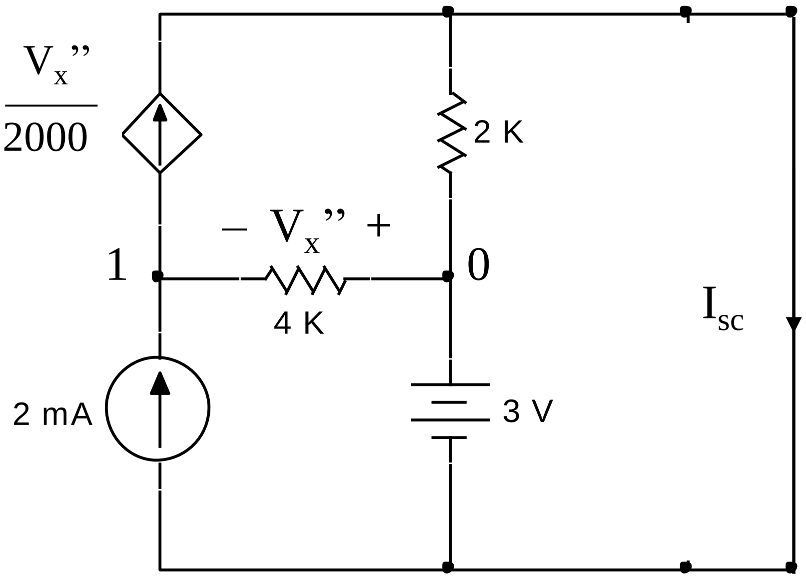
$$2 + V_x''/4 - V_x''/2 = 0$$

$$V_x'' = 8 \text{ V}$$

$$I_{sc} = I_{2K} + V_x''/2$$

$$= 3/2 \;+\; 8/2$$

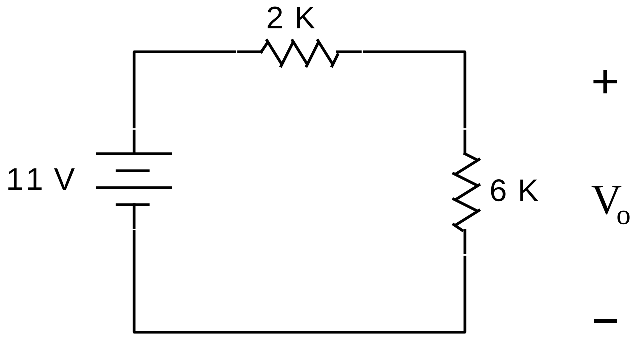
$$= 11/2 \text{ mA}$$



Solution

$R_{Th} = V_{oc} / I_{sc} = 11 / (11/2) = 2\ K$

Connect Thevenin's equivalent circuit to the remainder of the network, and determine V_o:

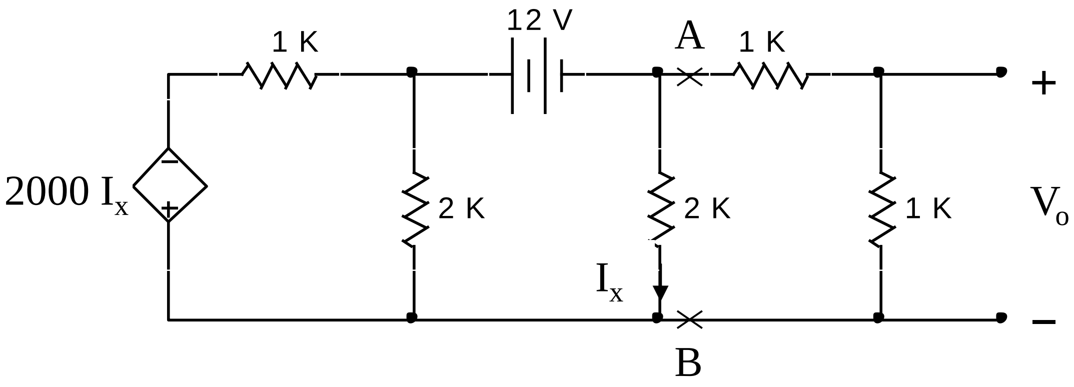


$V_o = 11*6 / (6+2)$

$= 33/4\ V$

Example #11

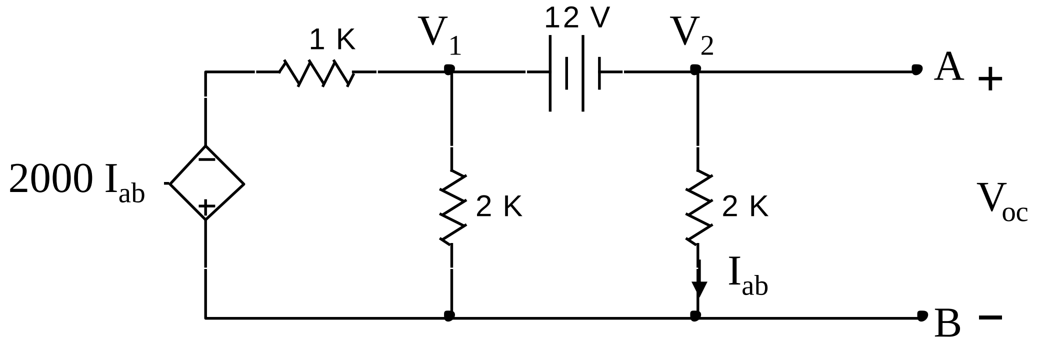
Use Thevenin's theorem to find V_o



Solution: Open the circuit at terminals A-B, and find

Thevenin's equivalent circuit between these terminals

Solution



Use nodal analysis to determine V_{oc}:

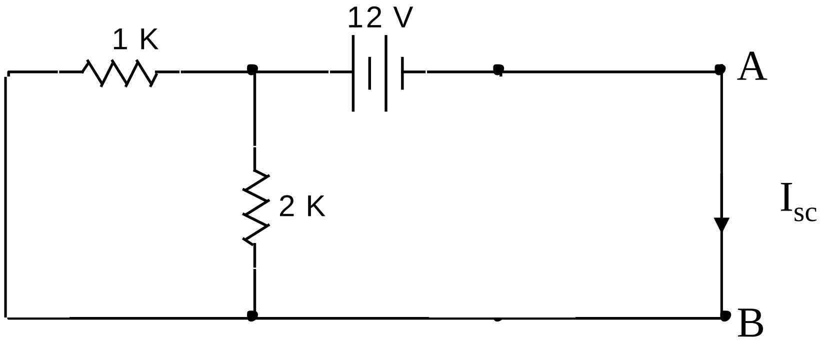
$V_1 – V_2 = 12$ (1)

$(V_1 – (– 2I_{ab}))/1 + V_1/2 + V_2/2$ (2)

$I_{ab} = V_2/2$ (3)

Solving the equations simultaneously yields: $V_2 = – 6$ V

Solution



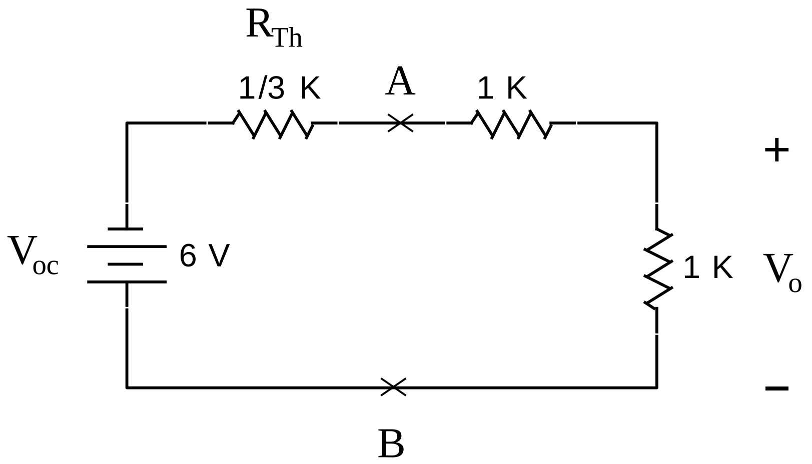
Short circuit $\rightarrow I_{ab} = 0 \quad \rightarrow 2000\ I_{ab} = 0$

Determine I_{sc}:

$I_{sc} = -12/(1//2) = -12/(2/3) = -18$ mA

$R_{Th} = V_{oc} / I_{sc} = (-6) / (-18) = 1/3$ K

Solution

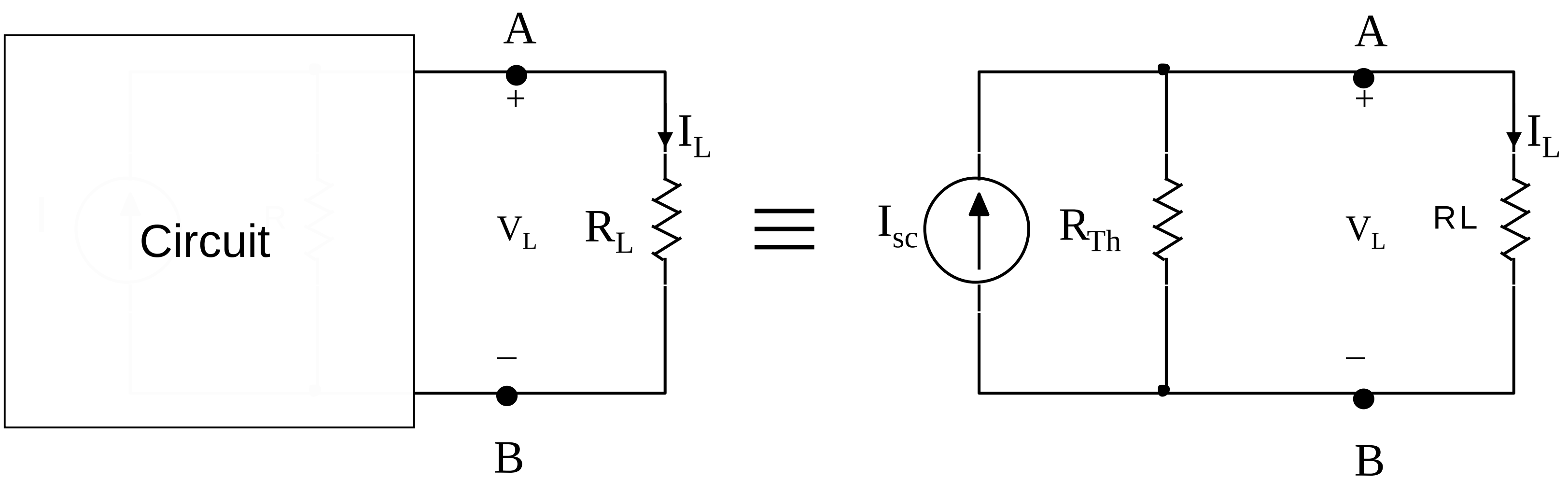


Connect Thevenin's equivalent circuit to the remainder of the network at terminals A-B, and determine V_o:

$$V_o = -6*1/(1+1+(1/3))$$

$$= -6/(7/3) = -2.57\ V$$

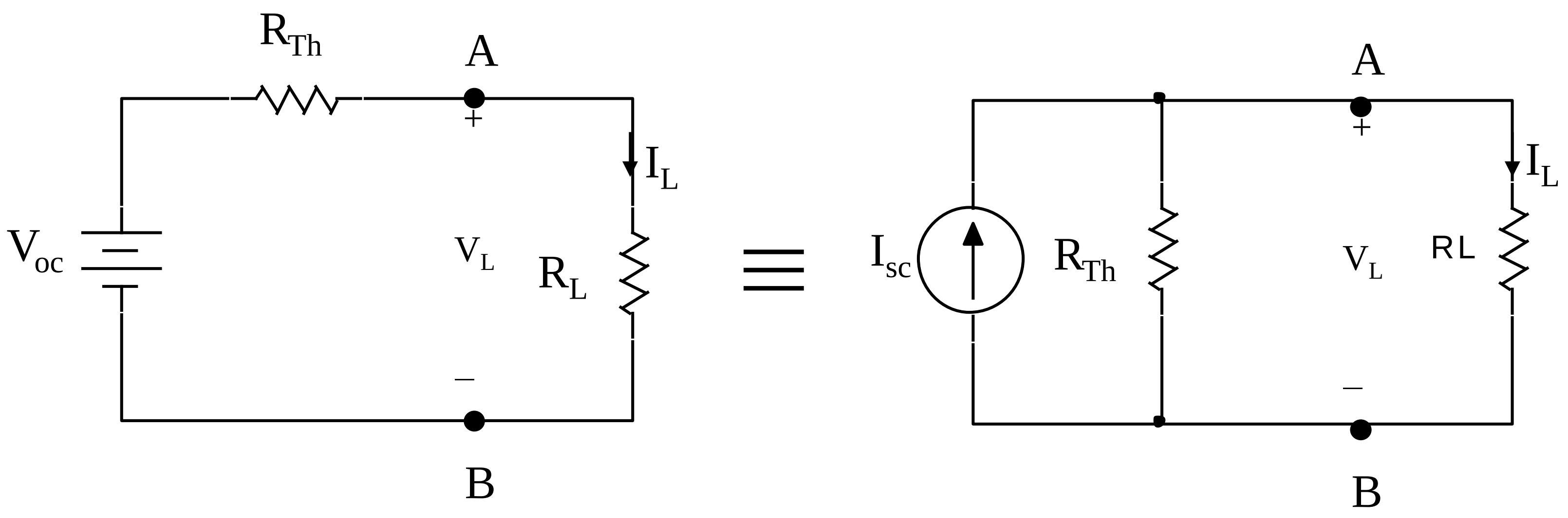
Norton's Theorem



I_{sc} : Short-circuit current between terminals A & B

R_{Th}: Equivalent resistance between terminals A &B with all independent sources set to zero

Thevenin's and Norton's Equivalent Circuits



Applying source transformation theorem:

$$V_{oc}/R_{Th} = I_{sc}$$

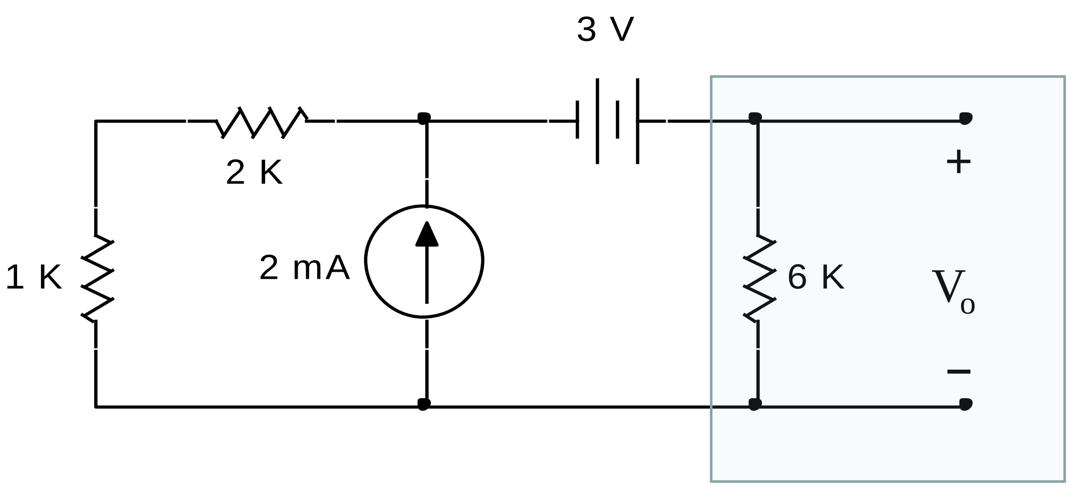
Applying Norton's Theorem

- Replace the load by a short circuit
- Find I_{sc} by applying all known circuit analysis techniques
- Determine the equivalent resistance of the circuit at the terminals (R_{Th})
- Connect the load to I_{sc} and R_{Th} and determine the required current or voltage.

Example #1

Find the voltage V_o using

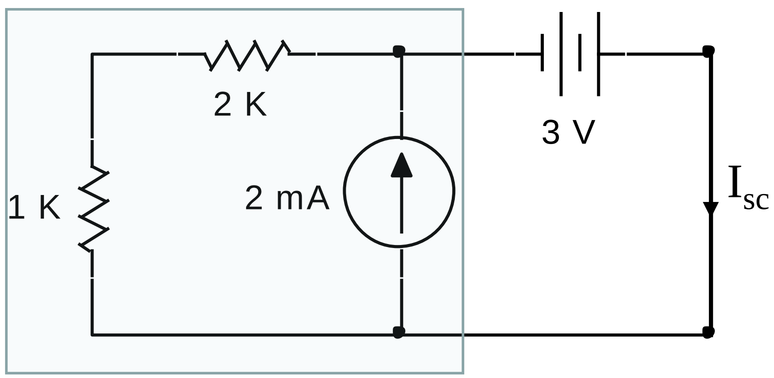
(b) Norton's theorem



Solution using Norton's Theorem

1- Short-circuit the load terminals

2- Calculate I_{sc}



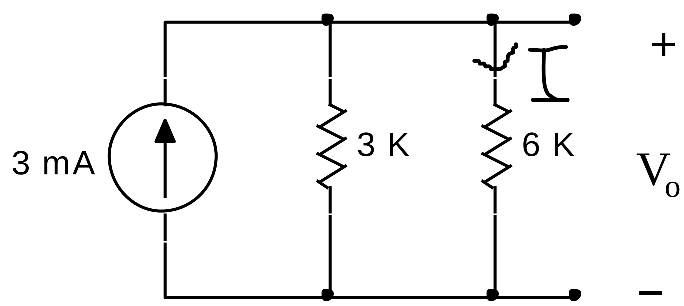
Using source transformation, we get a 6 V source in series with a resistor and the 3 V source →

$$I_{sc} = (3 + 6) / 3\ K = 3\ mA$$

Solution using Norton's Theorem

3- R_{Th} is calculated following the same procedure of Thevenin's theorem.

4- Connect the load to Norton's equivalent circuit, and calculate V_o



$$I = 3\left(\frac{3}{3+6}\right)$$

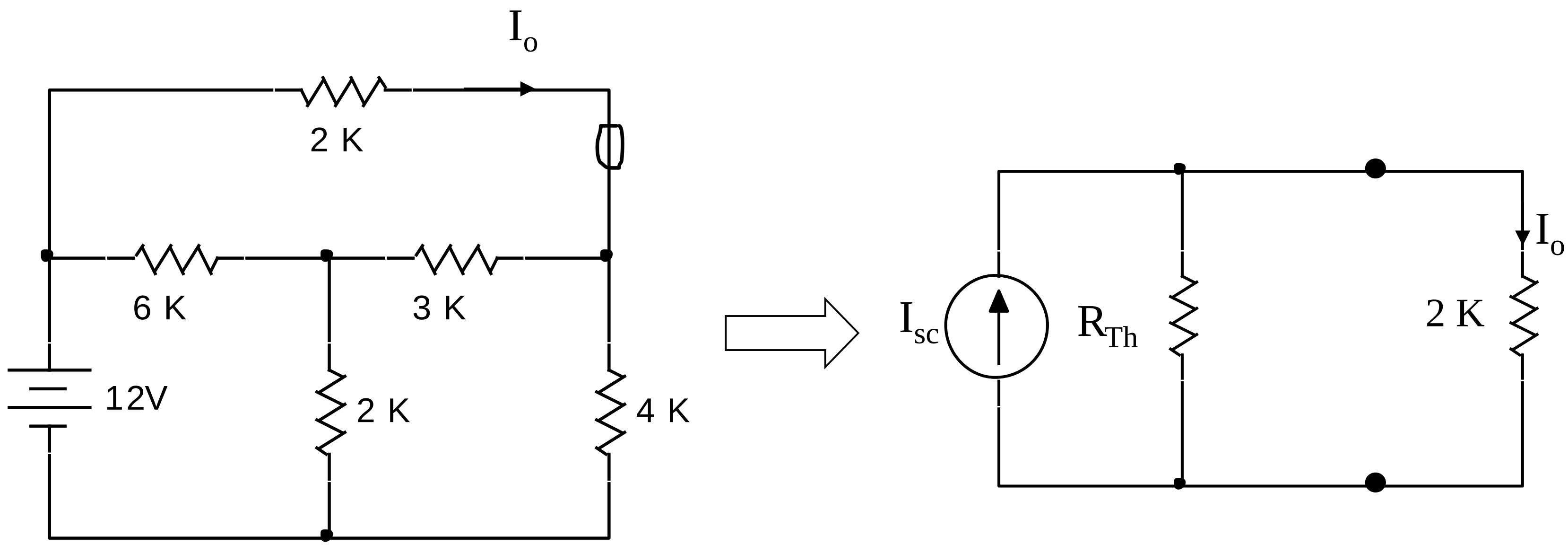
$$V_o = I \times 6$$

$$V_o = 6 \times 3\left(\frac{3}{3+6}\right)$$

V_o = 3 mA * (6//3) = 6 V

Example #2

Use Norton's theorem to find I_o



Solution

Using nodal analysis

$V_1 = 12$

Assuming that all currents are leaving node 2:

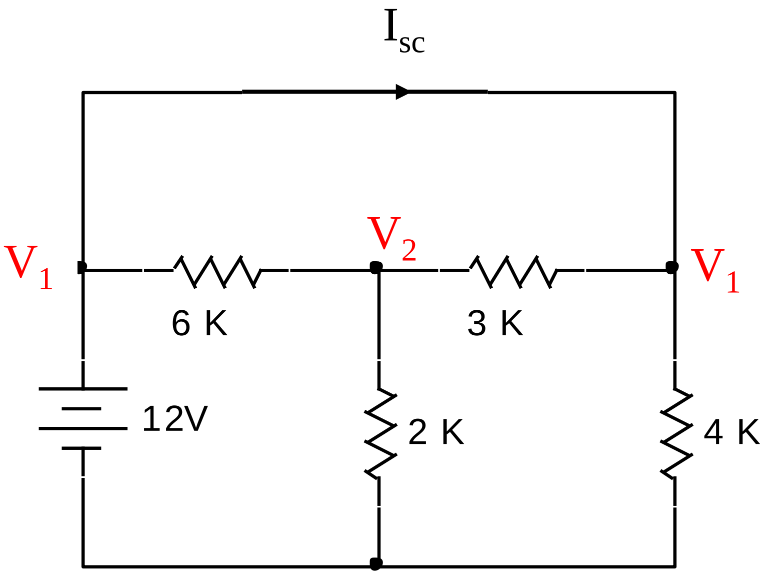
$(V_2 - 12)/6 + (V_2 - 12)/3 + V_2 /2 = 0$

$\rightarrow V_2 = 6$ V

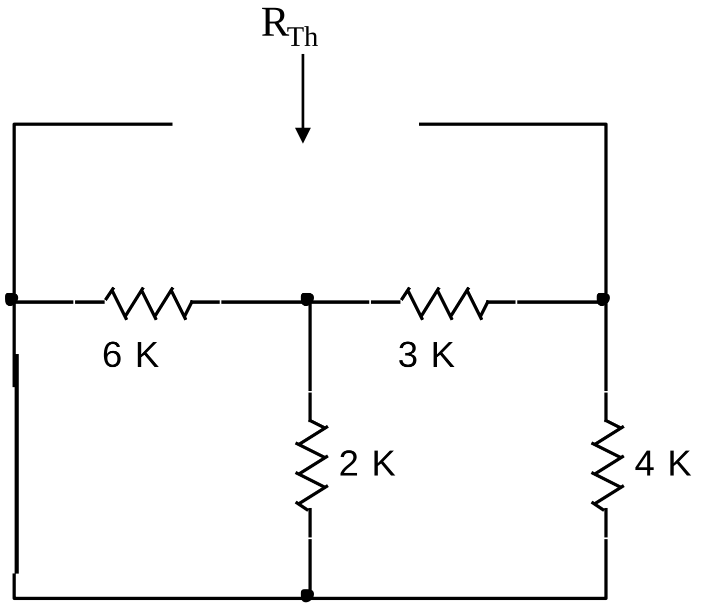
Applying KCL At node 1:

$I_{sc} = (12 - V_2) /3 + 12/4$

$= 5$ mA

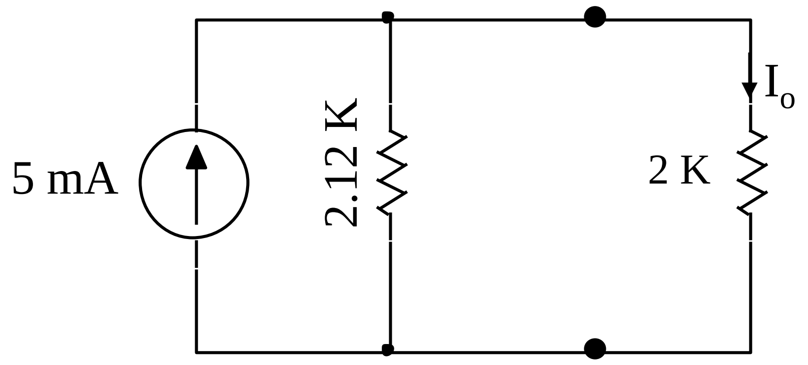


Solution



$$R_{Th} = 4 \,//\, (3+6//2)$$
$$= 2.12\ K$$

Solution



Connecting the load to Norton's equivalent circuit

$\rightarrow$ $I_o = 5 * 2.12 / (2.12+2)K$

$= 10.6 / 4.12\ K \quad = \ 2.57\ mA$

Exercises

Exercise#1

Example 4.8 Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals a - b. then find the current through $R_L = 6, 16,$ and $36\ \Omega$.

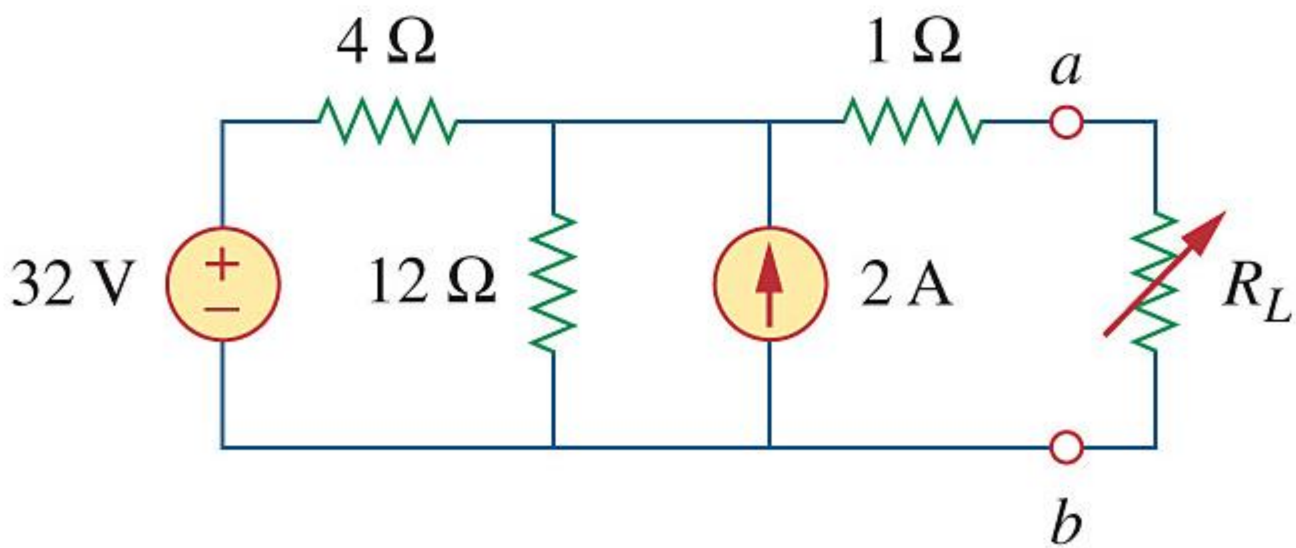


Figure 4.27

Solution :

Turn off all independent sources, the circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{Th} = 4 \,||\, 12 + 1 = \frac{4 \times 12}{4 + 12} + 1 = 4 \text{ (W)}$$

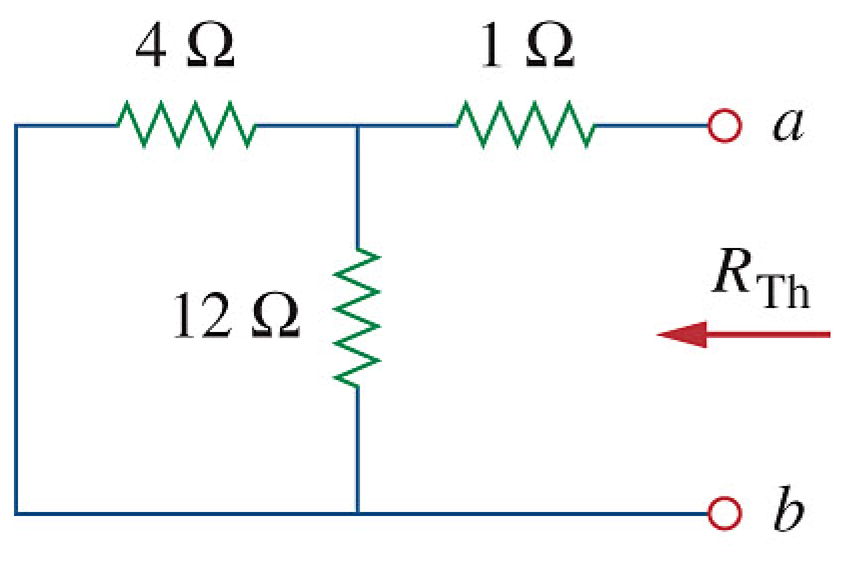


Figure 4.28(a)

Open-circuiting R_L, the circuit becomes the one in Fig. 4.28(b).

$$\frac{V_{Th}-32}{4}+\frac{V_{Th}}{12}-2=0 \Rightarrow V_{Th}=30\ \text{(V)}$$

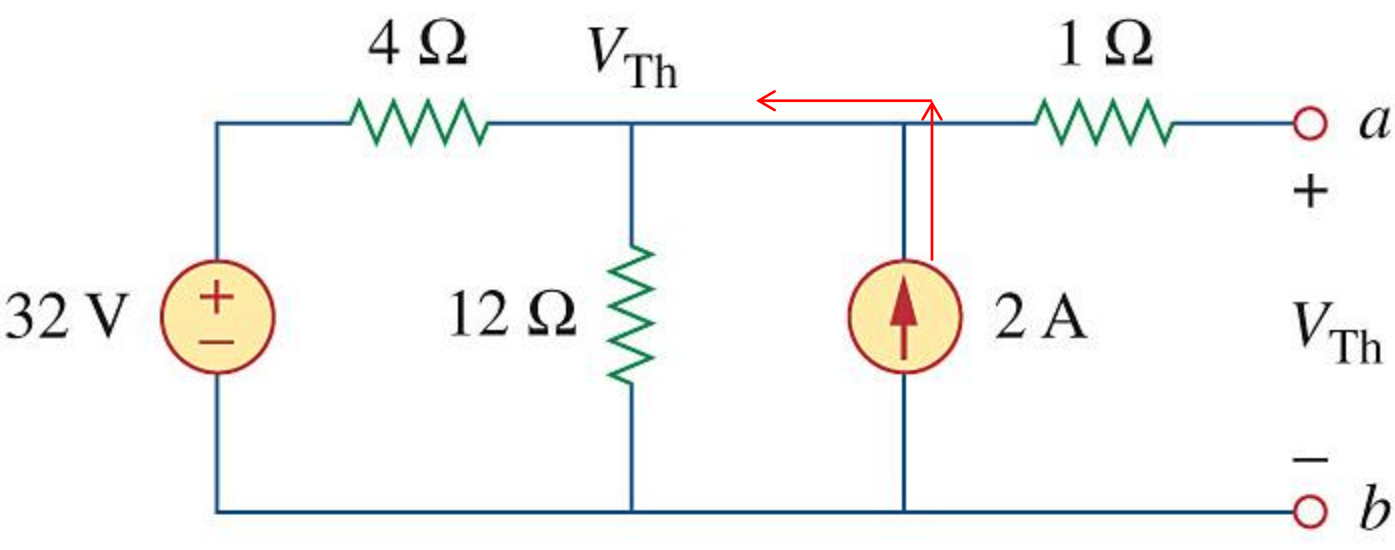


Figure 4.28(b)

The circuit in Fig. 4.27 can be replaced by the circuit shown in Fig. 4.29.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L} = 3, 1.5, 0.75\ (\mathrm{A})$$

when $R_L = 6, 16, 26\ \Omega.$

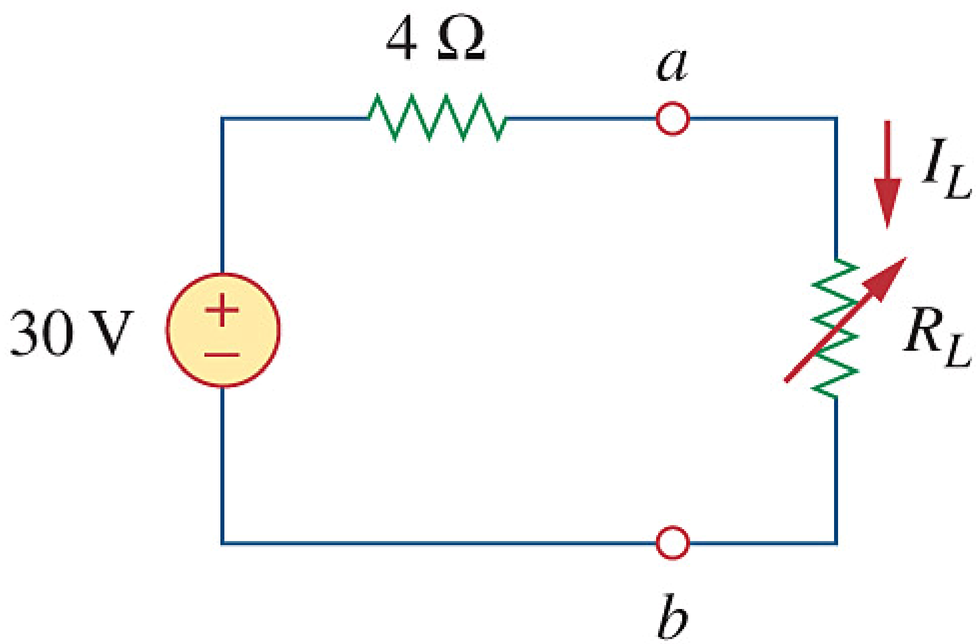


Figure 4.29

Exercise #2

Practice Problem 4.9 Find the Thevenin equivalent circuit of the circuit in Fig. 4.34 to the left of the terminals.

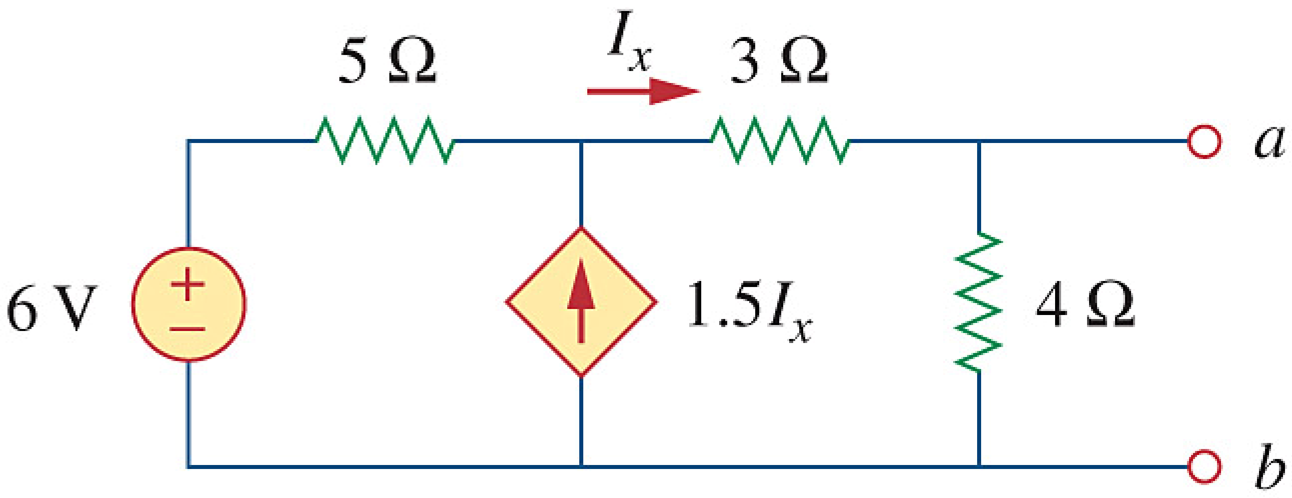


Figure 4.34

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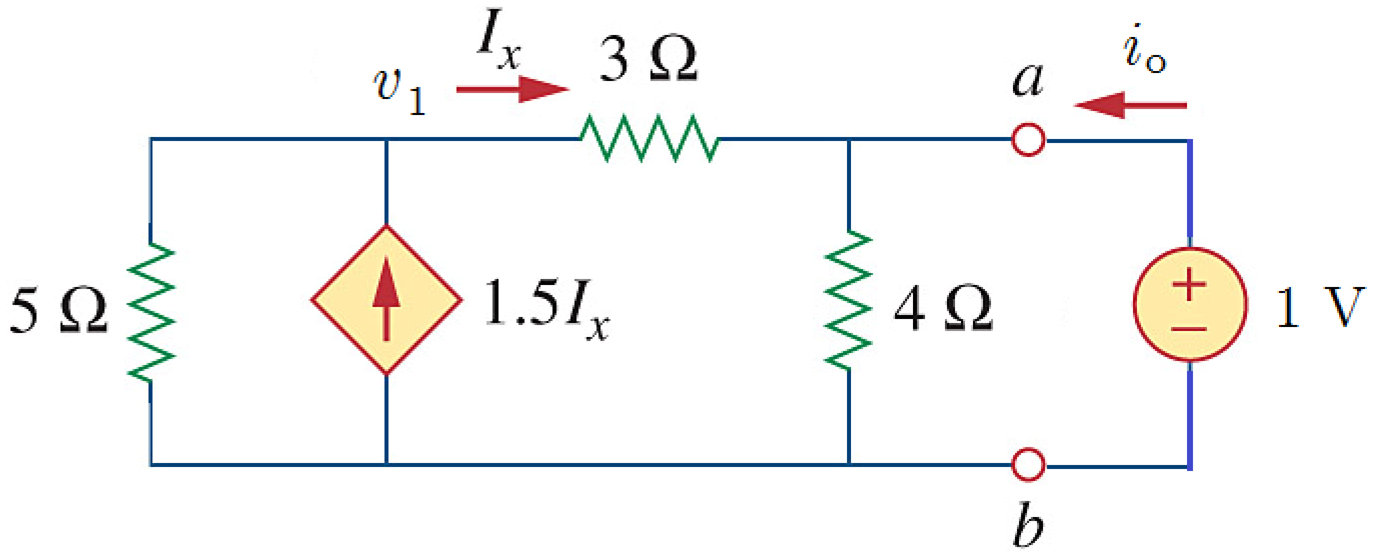
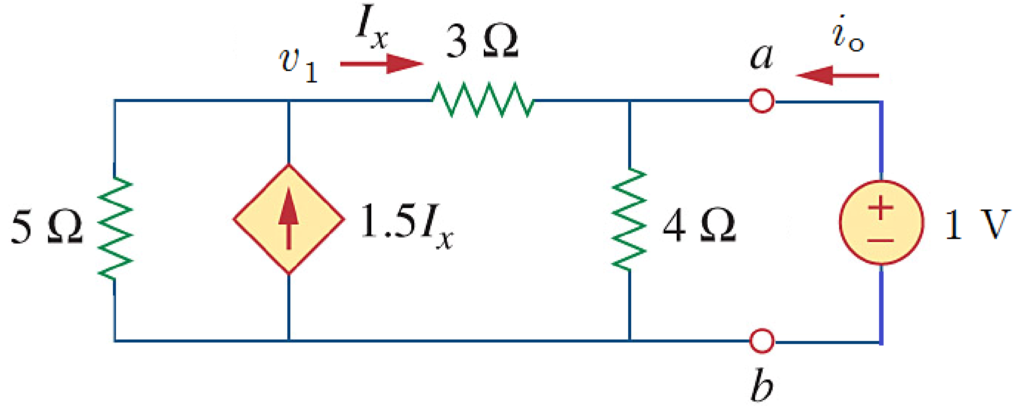
Solution :

To find R_{Th}, we turn off the independent voltage source and apply a test voltage source at the terminals a - b.



For Practice Problem 4.9: Find the Thevenin resistance



For Practice Problem 4.9: Find the Thevenin resistance

$$\begin{cases} I_x = \dfrac{v_1 - 1}{3} \\ \dfrac{v_1}{5} + I_x = 1.5 I_x \end{cases} \Rightarrow I_x = -2 \text{ (A)}$$

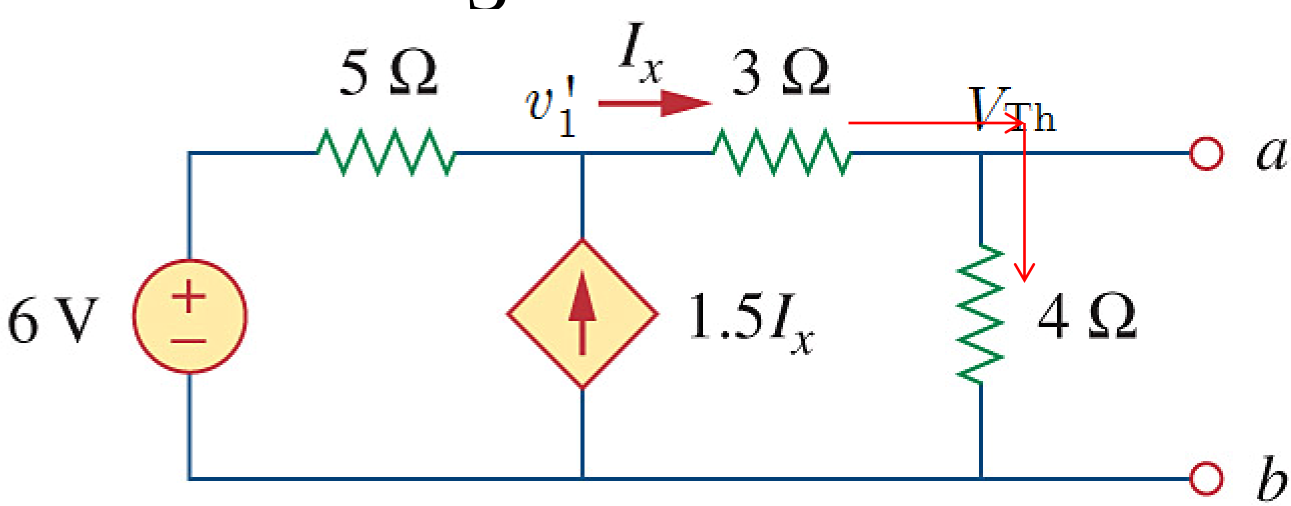
$$i_o = -I_x + \frac{1}{4} = \frac{9}{4} \text{ (A)}$$

$$R_{Th} = \frac{1}{i_o} = \frac{4}{9} \approx 0.44 \text{ (W)}$$

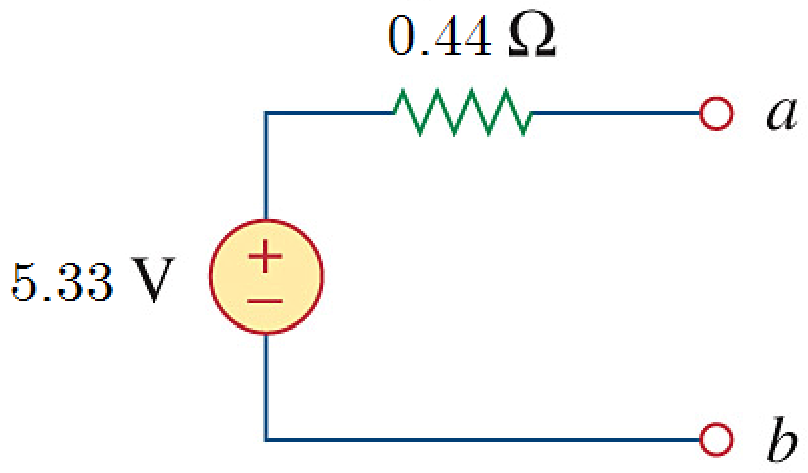
V_{Th} is the terminal voltage $V_{ab} = 4I_x$.

$$\begin{cases} I_x = \dfrac{v_1'}{3+4} \\ \dfrac{v_1' - 6}{5} + I_x = 1.5 I_x \end{cases} \Rightarrow I_x = \frac{4}{3}\ (\text{A})$$

$$V_{Th} = 4I_x = \frac{16}{3} \approx 5.33\ (\text{V})$$



For Practice Problem 4.9: Find the Thevenin voltage.



For Practice Problem 4.9:
The Thevenin Equivalent.

Exercise #3

Practice Problem 4.11 Find the Norton equivalent circuit for the circuit in Fig. 4.42, at terminals *a* - *b*.

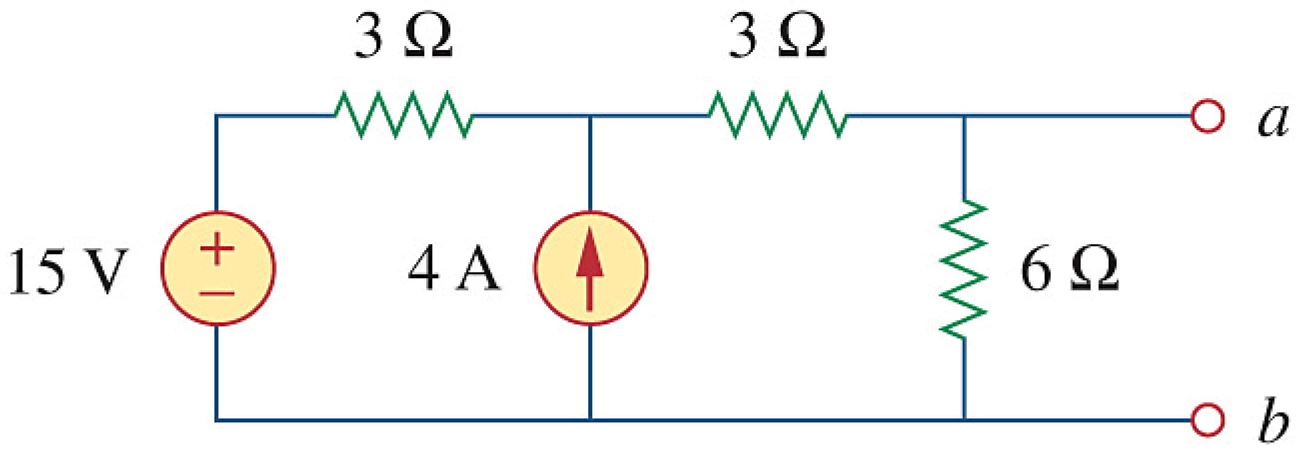
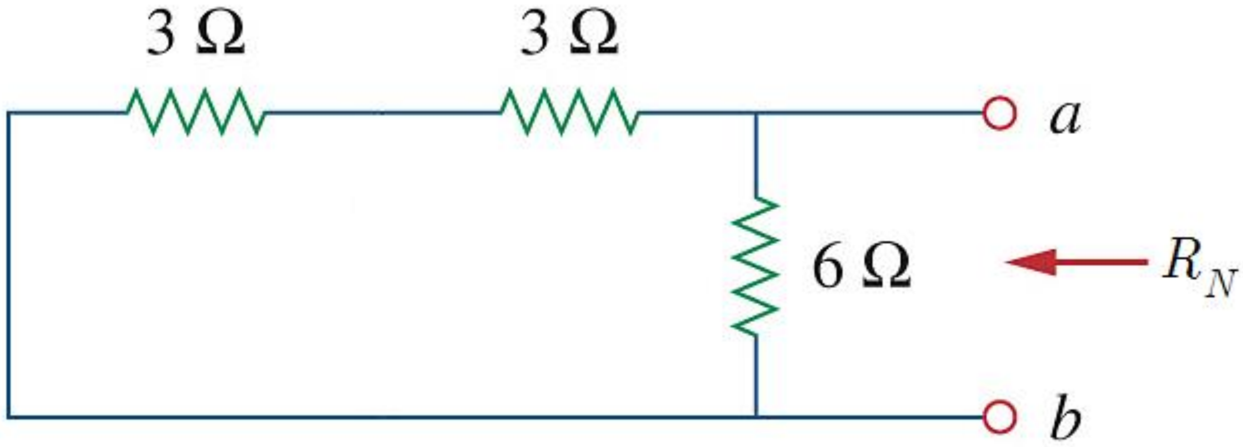


Figure 4.42

Solution :

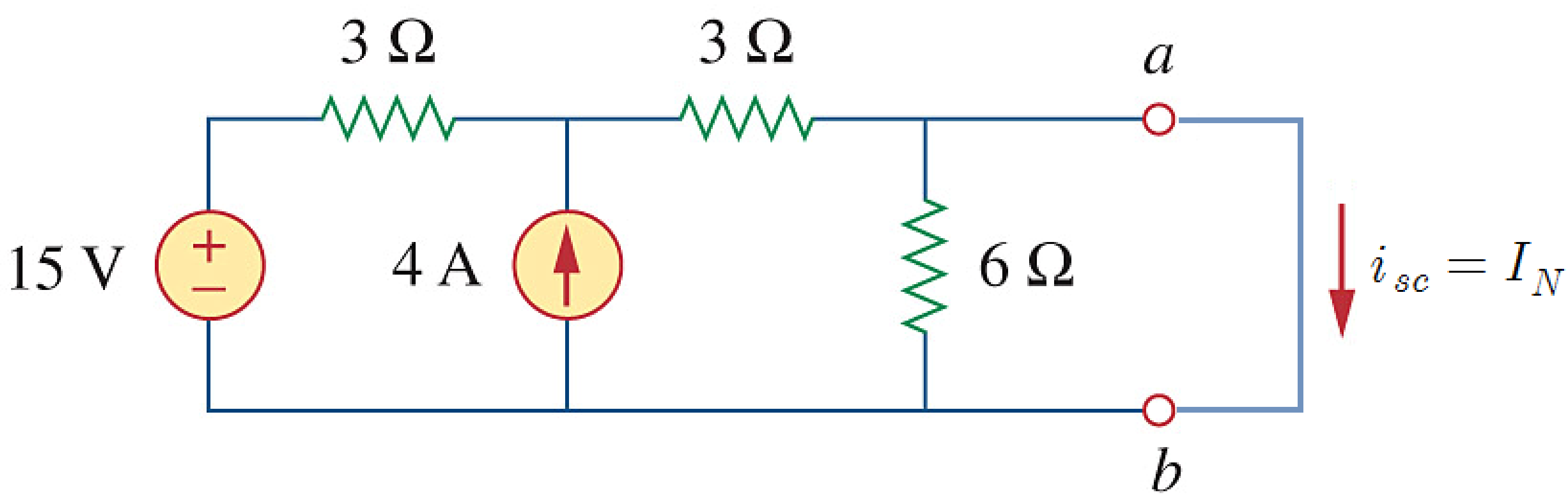
Turn off the voltage and current sources, the Norton resistance is

$$R_N = (3+3) \parallel 6 = 6 \parallel 6 = 3\ (\Omega)$$



Short-circuit terminals *a* and *b*, ignore the 6-Ω resistor, apply superposition principle,

$$I_N = I_N' + I_N'' = \frac{15}{3+3} + 4\times\frac{3}{3+3} = \frac{9}{2} = 4.5\ (\text{A})$$



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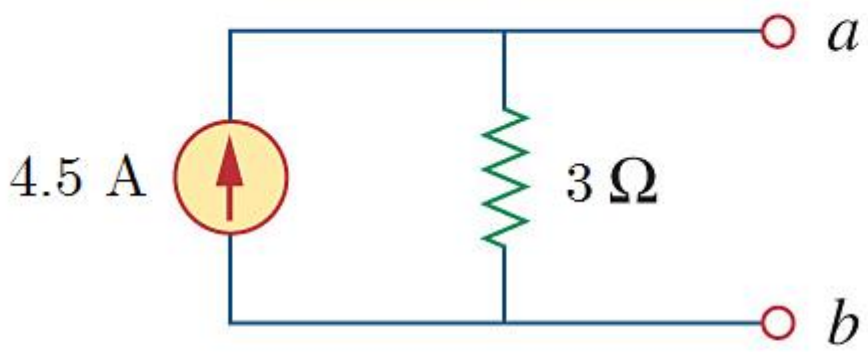
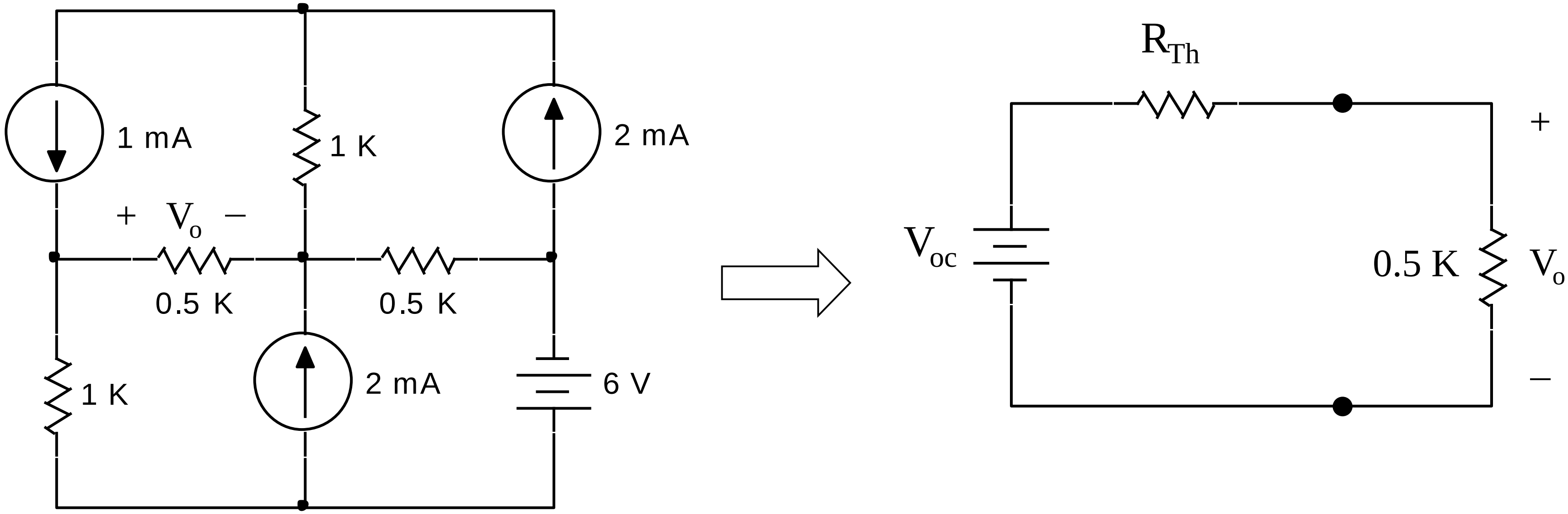
The Norton equivalent is shown below.



Exercise #4

Use Thevenin's theorem to find V_o



Solution

$V_{oc} = V_1 – V_2$

$V_1 = 1\text{ K} * 1\text{ mA} = 1\text{ V}$

Applying KCL at node 4:

$I_1 = 2\text{ mA} – 1\text{ mA} = 1\text{ mA}$

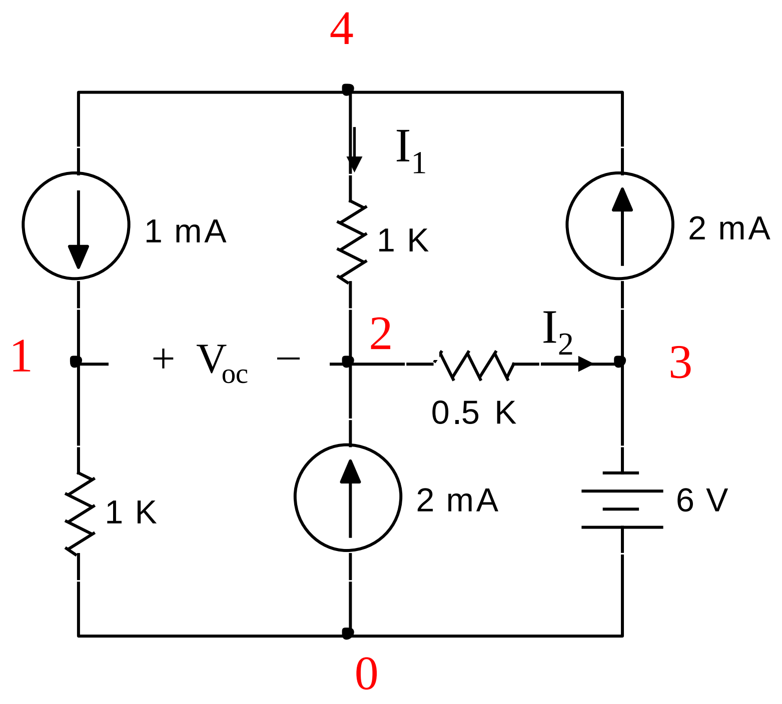
Applying KCL at node 2:

$I_2 = 1\text{ mA} + 2\text{ mA} = 3\text{ mA}$

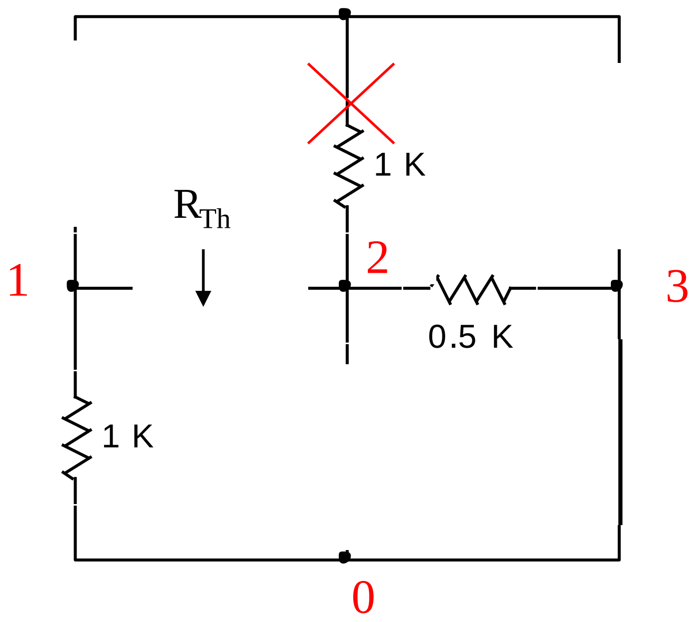
$V_2 = 0.5 * I_2 – 6 = – 4.5\text{ V}$

$V_{oc} = V_1 – V_2$

$= 1 – (– 4.5)) = 5.5\text{ V}$

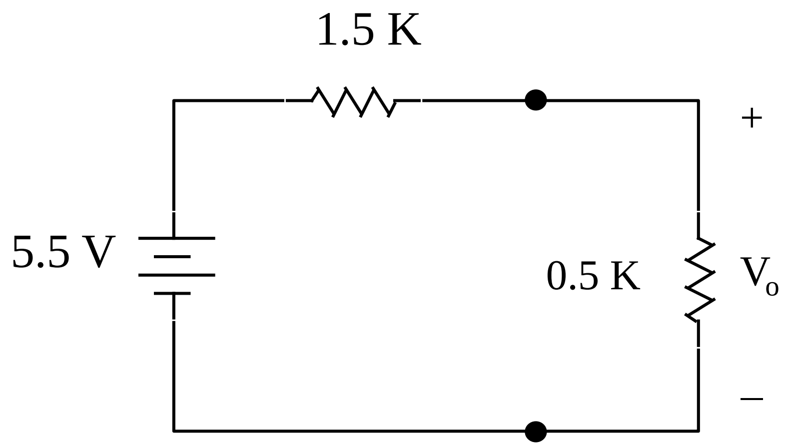


Solution



$$R_{Th} = 1\text{ K} + 0.5\text{ K}$$
$$= 1.5\text{ K}$$

Solution



Connecting the load to Thevenin's equivalent circuit

→ V_o = 5.5*0.5 / (1.5+0.5)K

= 2.75 / 2 K = 1.375 V